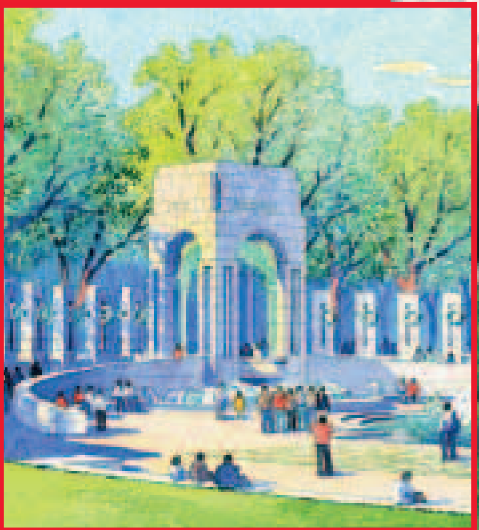
UNIT 4

Area and Volume

Area and volume can be used to analyze real-world situations. In this unit, you will learn about formulas used to find the areas of two-dimensional figures and the surface areas and volumes of three-dimensional figures.



(l)Courtesy National World War II Memorial, (r)Rob Crandall/Stock Boston

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WebQuest Internet Project

Town With Major D-Day Losses Gets Memorial

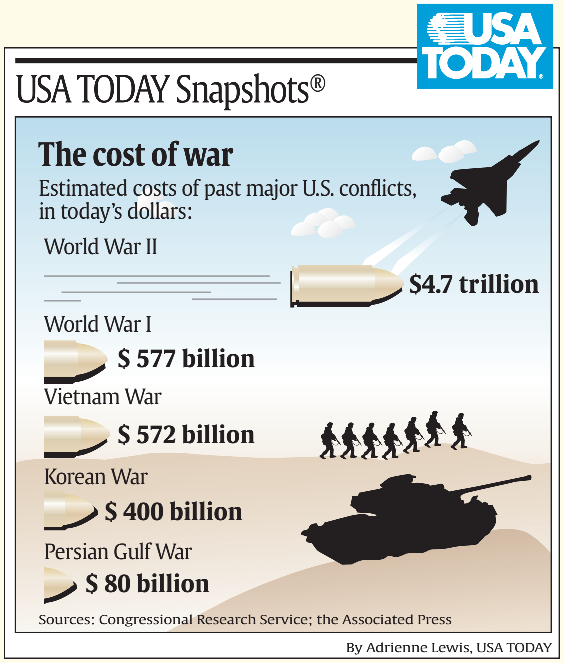
Source: *USA TODAY,* May 27, 2001

"BEDFORD, Va. For years, World War II was a sore subject that many families in this small farming community avoided. 'We lost so many men,' said Boyd Wilson, 79, who joined Virginia's 116th National Guard before it was sent to war. 'It was just painful.' The war hit Bedford harder than perhaps any other small town in America, taking 19 of its sons, fathers and brothers in the opening moments of the Allied invasion of Normandy. Within a week, 23 of Bedford's 35 soldiers were dead. It was the highest per capita loss for any U.S. community." In this project, you will use scale drawings, surface area, and volume to design a memorial to honor war veterans.

Log on to www.geometryonline.com/webquest. Begin your WebQuest by reading the Task.

Continue working on your WebQuest as you study Unit 4.

Lesson	11-4	12-5	13-3
Page	618	662	703



Chapter 11

Areas of Polygons and Circles

What You'll Learn

- **Lessons 11-1, 11-2, and 11-3** Find areas of parallelograms, triangles, rhombi, trapezoids, regular polygons, and circles.
- **Lesson 11-4** Find areas of irregular figures.
- **Lesson 11-5** Find geometric probability and areas of sectors and segments of circles.

Key Vocabulary

- **apothem** (p. 610)
- **irregular figure** (p. 617)
- **geometric probability** (p. 622)
- **sector** (p. 623)
- **segment** (p. 624)

Why It's Important

Skydivers use geometric probability when they attempt to land on a target marked on the ground. They can determine the chances of landing in the center of the target. *You will learn about skydiving in Lesson 11-5.*

Ken Fisher/Getty Images

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Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 11.

For Lesson 11-1 — Area of a Rectangle

The area and width of a rectangle are given. Find the length of the rectangle. *(For review, see pages 732–733.)*

1. $A = 150, w = 15$ **2.** $A = 38, w = 19$ **3.** $A = 21.16, w = 4.6$

4. $A = 2000, w = 32$ **5.** $A = 450, w = 25$ **6.** $A = 256, w = 20$

For Lessons 11-2 and 11-4 — Evaluate a Given Expression

Evaluate each expression if $a = 6$, $b = 8$, $c = 10$, and $d = 11$. *(For review, see page 736.)*

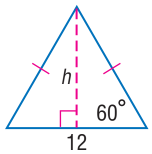
7. $\frac{1}{2}a(b + c)$ **8.** $\frac{1}{2}ab$ **9.** $\frac{1}{2}(2b + c)$

10. $\frac{1}{2}d(a + c)$ **11.** $\frac{1}{2}(b + c)$ **12.** $\frac{1}{2}cd$

For Lesson 11-3 — Height of a Triangle

Find h in each triangle. *(For review, see Lesson 7-3.)*

13.



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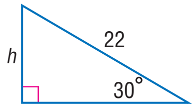

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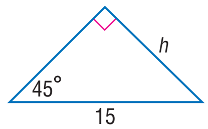
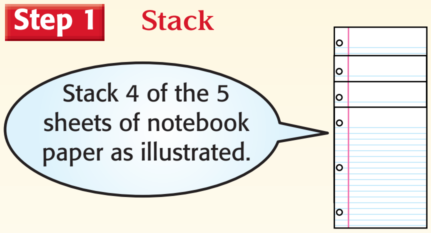


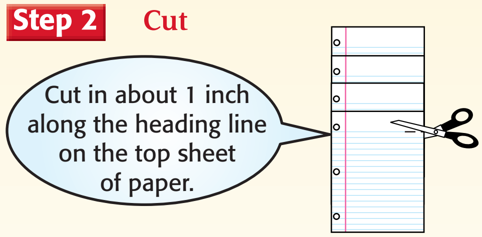
FOLDABLES™ Study Organizer

Areas of Polygons and Circles Make this Foldable to help you organize your notes about areas of polygons and circles. Begin with five sheets of notebook paper.

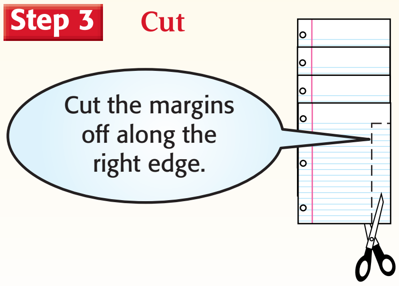
Step 1 Stack



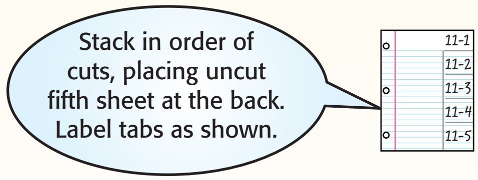
Step 2 Cut



Step 3 Cut



Step 4 Stack



Reading and Writing As you read and study the chapter, take notes and record examples of areas of polygons and circles.

Reading Mathematics

Prefixes

Many of the words used in mathematics use the same prefixes as other everyday words. Understanding the meaning of the prefixes can help you understand the terminology better.

Prefix	Meaning	Everyday Words	Meaning
bi-	2	bicycle	a 2-wheeled vehicle
		bipartisan	involving members of 2 political parties
tri-	3	triangle	closed figure with 3 sides
		tricycle	a 3-wheeled vehicle
		triplet	one of 3 children born at the same time
quad-	4	quadrilateral	closed figure with 4 sides
		quadriceps	muscles with 4 parts
		quadruple	four times as many
penta-	5	pentagon	closed figure with 5 sides
		pentathlon	athletic contest with 5 events
hexa-	6	hexagon	closed figure with 6 sides
hept-	7	heptagon	closed figure with 7 sides
oct-	8	octagon	closed figure with 8 sides
		octopus	animal with 8 legs
dec-	10	decagon	closed figure with 10 sides
		decade	a period of 10 years
		decathlon	athletic contest with 10 events

Several pairs of words in the chart have different prefixes, but the same root word. *Pentathlon* and *decathlon* are both athletic contests. *Heptagon* and *octagon* are both closed figures. Knowing the meaning of the root of the term as well as the prefix can help you learn vocabulary.

Reading to Learn

Use a dictionary to find the meanings of the prefix and root for each term. Then write a definition of the term.

1. bisector
2. polygon
3. equilateral
4. concentric
5. circumscribe
6. collinear

7. RESEARCH Use a dictionary to find the meanings of the prefix and root of *circumference.*

8. RESEARCH Use a dictionary or the Internet to find as many words as you can with the prefix *poly-* and the definition of each.

11-1 Areas of Parallelograms

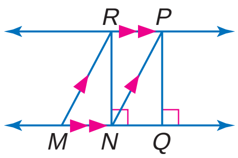
What You'll Learn

- Find perimeters and areas of parallelograms.
- Determine whether points on a coordinate plane define a parallelogram.

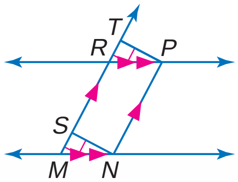
How is area related to garden design?

This composition of square-cut granite and moss was designed by Shigemori Mirei in Kyoto, Japan. How could you determine how much granite was used in this garden?

AREAS OF PARALLELOGRAMS Recall that a *parallelogram* is a quadrilateral with both pairs of opposite sides parallel. Any side of a parallelogram can be called a base. For each base, there is a corresponding altitude that is perpendicular to the base.



In ▱*MNPR*, if $\overline{MN}$ is the base, $\overline{RN}$ and $\overline{PQ}$ are altitudes. The length of an altitude is called the *height* of the parallelogram. If $\overline{MR}$ is the base, then the altitudes are $\overline{PT}$ and $\overline{NS}$.

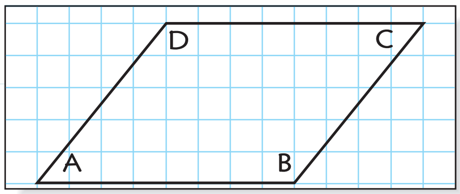


Geometry Activity

Area of a Parallelogram

Model

- Draw a parallelogram with a base 8 units long and an altitude of 5 units on grid paper. Label the vertices on the interior of the angles with letters *A, B, C,* and *D*.
- Fold ▱*ABCD* so that *A* lies on *B* and *C* lies on *D*, forming a rectangle.



Analyze

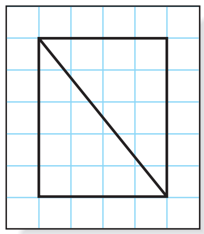
1. What is the area of the rectangle?

2. How many rectangles form the parallelogram?

3. What is the area of the parallelogram?

4. How do the base and altitude of the parallelogram relate to the length and width of the rectangle?

5. Make a conjecture Use what you observed to write a formula for the area of a parallelogram.



Michael S. Yamashita/CORBIS

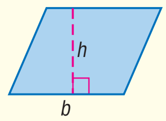
The Geometry Activity leads to the formula for the area of a parallelogram.

Study Tip

Units
Length is measured in linear units, and area is measured in square units.

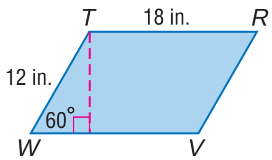
Key Concept — *Area of a Parallelogram*

If a parallelogram has an area of A square units, a base of b units, and a height of h units, then $A = bh$.



Example 1 Perimeter and Area of a Parallelogram

Find the perimeter and area of ▱*TRVW*.



Base and Side: Each pair of opposite sides of a parallelogram has the same measure. Each base is 18 inches long, and each side is 12 inches long.

Study Tip

Look Back
To review **perimeter of polygons**, see Lesson 1-6.

Perimeter: The perimeter of a polygon is the sum of the measures of its sides. So, the perimeter of ▱*TRVW* is 2(18) + 2(12) or 60 inches.

Height: Use a 30°-60°-90° triangle to find the height. Recall that if the measure of the leg opposite the 30° angle is x, then the length of the hypotenuse is $2x$, and the length of the leg opposite the 60° angle is $x\sqrt{3}$.

$12 = 2x$ Substitute 12 for the hypotenuse.

$6 = x$ Divide each side by 2.

So, the height of the parallelogram is $x\sqrt{3}$ or $6\sqrt{3}$ inches.

Area:

$A = bh$ Area of a parallelogram

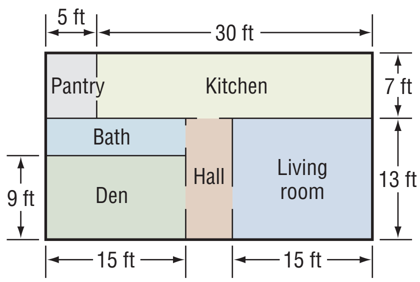
$= 18(6\sqrt{3})$ $b = 18, h = 6\sqrt{3}$

$= 108\sqrt{3}$ or about 187.1

The perimeter of ▱*TRVW* is 60 inches, and the area is about 187.1 square inches.

Example 2 Use Area to Solve a Real-World Problem

INTERIOR DESIGN **The Waroners are planning to recarpet part of the first floor of their house. Find the amount of carpeting needed to cover the living room, den, and hall.**



To estimate how much they can spend on carpeting, they need to find the square yardage of each room.

Living Room: $w = 13$ ft, $\ell = 15$ ft

Den: $w = 9$ ft, $\ell = 15$ ft

Hall: It is the same width as the living room, so $w = 13$. The total length of the house is 35 feet. So, $\ell = 35 - 15 - 15$ or 5 feet.

Area

Living Room	Den	Hall
$A = \ell w$	$A = \ell w$	$A = \ell w$
$= 13 \cdot 15$	$= 9 \cdot 15$	$= 5 \cdot 13$
$= 195\text{ ft}^2$	$= 135\text{ ft}^2$	$= 65\text{ ft}^2$

The total area is 195 + 135 + 65 or 395 square feet. There are 9 square feet in one square yard, so divide by 9 to convert from square feet to square yards.

$$395 \text{ ft}^2 \div \frac{9 \text{ ft}^2}{1 \text{ yd}^2} = 395 \text{ ft}^2 \times \frac{1 \text{ yd}^2}{9 \text{ ft}^2}$$
$$\approx 43.9 \text{ yd}^2$$

Therefore, 44 square yards of carpeting are needed to cover these areas.

PARALLELOGRAMS ON THE COORDINATE PLANE Recall the properties of quadrilaterals that you studied in Chapter 8. Using these properties as well as the formula for slope and the Distance Formula, you can find the areas of quadrilaterals on the coordinate plane.

Study Tip

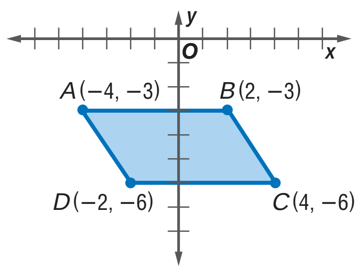
Look Back
To review **properties of parallelograms**, **rectangles**, and **squares**, see Lessons 8-3, 8-4, and 8-5.

Example 3 Area on the Coordinate Plane

COORDINATE GEOMETRY **The vertices of a quadrilateral are $A(-4, -3)$, $B(2, -3)$, $C(4, -6)$, and $D(-2, -6)$.**

a. Determine whether the quadrilateral is a *square*, a *rectangle*, or a *parallelogram*.

First graph each point and draw the quadrilateral. Then determine the slope of each side.



$$\text{slope of } \overline{AB} = \frac{-3-(-3)}{-4-2} = \frac{0}{-6} \text{ or } 0$$

$$\text{slope of } \overline{CD} = \frac{-6-(-6)}{4-(-2)} = \frac{0}{6} \text{ or } 0$$

$$\text{slope of } \overline{BC} = \frac{-3-(-6)}{2-4} = \frac{3}{-2}$$

$$\text{slope of } \overline{AD} = \frac{-3-(-6)}{-4-(-2)} = \frac{3}{-2}$$

Opposite sides have the same slope, so they are parallel. *ABCD* is a parallelogram. The slopes of the consecutive sides are *not* negative reciprocals of each other, so the sides are not perpendicular. Thus, the parallelogram is neither a square nor a rectangle.

b. Find the area of quadrilateral *ABCD*.

Base: $\overline{CD}$ is parallel to the x-axis, so subtract the x-coordinates of the endpoints to find the length: $CD = |4-(-2)|$ or 6.

Height: Since $\overline{AB}$ and $\overline{CD}$ are horizontal segments, the distance between them, or the height, can be measured on any vertical segment. Reading from the graph, the height is 3.

$A = bh$	Area formula
$= 6(3)$	$b = 6, h = 3$
$= 18$	Simplify.

The area of $\square ABCD$ is 18 square units.

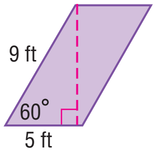
Check for Understanding

Concept Check

1. **Compare and contrast** finding the area of a rectangle and the area of a parallelogram.
2. **OPEN ENDED** Make and label a scale drawing of your bedroom. Then find its area in square yards.

Guided Practice

Find the perimeter and area of each parallelogram. Round to the nearest tenth if necessary.

3.


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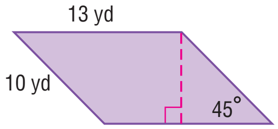

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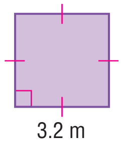
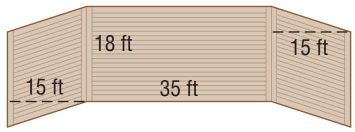


Given the coordinates of the vertices of quadrilateral *TVXY*, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of *TVXY*.

6. $T(0, 0)$, $V(2, 6)$, $X(6, 6)$, $Y(4, 0)$
7. $T(10, 16)$, $V(2, 18)$, $X(-3, -2)$, $Y(5, -4)$

Application

8. **DESIGN** Mr. Kang is planning to stain his deck. To know how much stain to buy, he needs to find the area of the deck. What is the area?



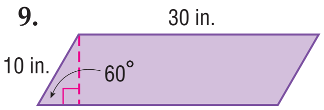
Practice and Apply

Homework Help

For Exercises	See Examples
9–14	1
15–17, 27, 28, 31	2
20–25	3

Extra Practice
See page 776.

Find the perimeter and area of each parallelogram. Round to the nearest tenth if necessary.

9.


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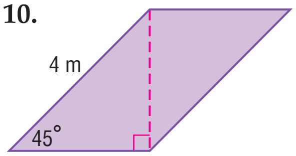

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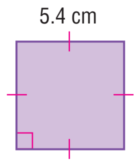

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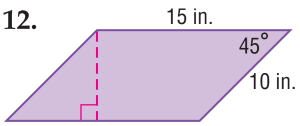

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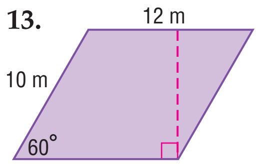

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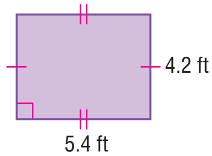

Find the area of each shaded region. Round to the nearest tenth if necessary.

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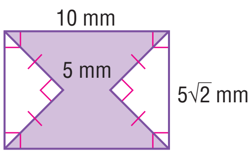

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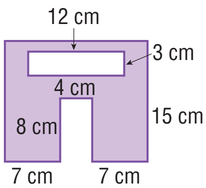

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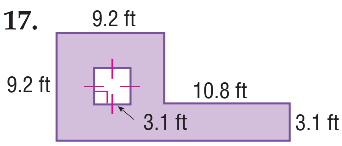
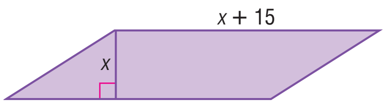
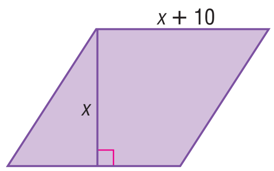


Find the height and base of each parallelogram given its area.

18. 100 square units



19. 2000 square units



CONTENTS

COORDINATE GEOMETRY **Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral.**

20. $A(0, 0), B(4, 0), C(5, 5), D(1, 5)$

21. $E(-5, -3), F(3, -3), G(5, 4), H(-3, 4)$

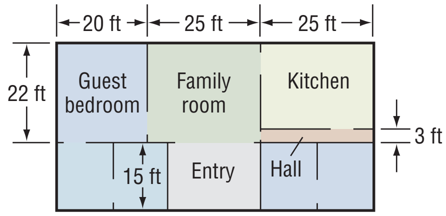
22. $J(-1, -4), K(4, -4), L(6, 6), M(1, 6)$

23. $N(-6, 2), O(2, 2), P(4, -6), Q(-4, -6)$

24. $R(-2, 4), S(8, 4), T(8, -3), U(-2, -3)$

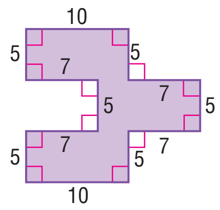
25. $V(1, 10), W(4, 8), X(2, 5), Y(-1, 7)$

26. **INTERIOR DESIGN** The Bessos are planning to have new carpet installed in their guest bedroom, family room, and hallway. Find the number of square yards of carpet they should order.



Find the area of each figure.

27.



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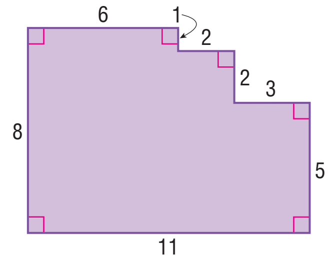


More About. . .

Art

A triptych originally referred to a Roman writing tablet with three panels that were hinged together.

Source: www.artlex.com

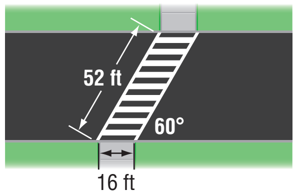
ART **For Exercises 29 and 30, use the following information.**
A *triptych* painting is a series of three pieces with a similar theme displayed together. Suppose the center panel is a 12-inch square and the panels on either side are 12 inches by 5 inches. The panels are 2 inches apart with a 3 inch wide border around the edges.

Study for a Triptych, by Albert Gleizes

29. Determine whether the triptych will fit a 45-inch by 20-inch frame. Explain.

30. Find the area of the artwork.

31. **CROSSWALKS** A crosswalk with two stripes each 52 feet long is at a 60° angle to the curb. The width of the crosswalk at the curb is 16 feet. Find the perpendicular distance between the stripes of the crosswalk.



VARYING DIMENSIONS **For Exercises 32–34, use the following information.**
A parallelogram has a base of 8 meters, sides of 11 meters, and a height of 10 meters.

32. Find the perimeter and area of the parallelogram.

33. Suppose the dimensions of the parallelogram were divided in half. Find the perimeter and the area.

34. Compare the perimeter and area of the parallelogram in Exercise 33 with the original.

35. **CRITICAL THINKING** A piece of twine 48 inches long is cut into two lengths. Each length is then used to form a square. The sum of the areas of the two squares is 74 square inches. Find the length of each side of the smaller square and the larger square.

36. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

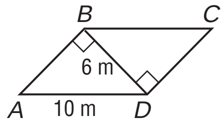
How is area related to garden design?

Include the following in your answer:

- how to determine the total area of granite squares, and
- other uses for area.

Standardized Test Practice

37. What is the area of ▱$ABCD$?



Ⓐ 24 m^2 Ⓑ 30 m^2 Ⓒ 48 m^2 Ⓓ 60 m^2

38. **ALGEBRA** Which statement is correct?

Ⓐ $x^2 > (x - 1)^2$ Ⓒ $x^2 < (x - 1)^2$

Ⓑ $x^2 = (x - 1)^2$ Ⓓ The relationship cannot be determined.

Maintain Your Skills

Mixed Review

Determine the coordinates of the center and the measure of the radius for each circle with the given equation. *(Lesson 10-8)*

39. $(x - 5)^2 + (y - 2)^2 = 49$

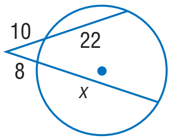
40. $(x + 3)^2 + (y + 9)^2 - 81 = 0$

41. $\left(x + \frac{2}{3}\right)^2 + \left(y - \frac{1}{9}\right)^2 - \frac{4}{9} = 0$

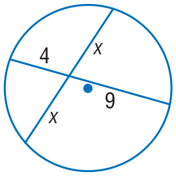
42. $(x - 2.8)^2 + (y + 7.6)^2 = 34.81$

Find x. Assume that segments that appear to be tangent are tangent. *(Lesson 10-7)*

43.



44.



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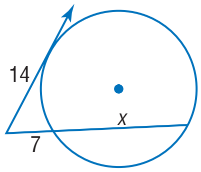


COORDINATE GEOMETRY **Draw the rotation image of each triangle by reflecting the triangles in the given lines. State the coordinates of the rotation image and the angle of rotation.** *(Lesson 9-3)*

46. $\triangle ABC$ with vertices $A(-1, 3)$, $B(-4, 6)$, and $C(-5, 1)$, reflected in the y-axis and then the x-axis

47. $\triangle FGH$ with vertices $F(0, 4)$, $G(-2, 2)$, and $H(2, 2)$, reflected in $y = x$ and then the y-axis

48. $\triangle LMN$ with vertices $L(2, 0)$, $M(3, -3)$, and $N(1, -4)$, reflected in the y-axis and then the line $y = -x$

49. **BIKES** Nate is making a ramp for bike jumps. The ramp support forms a right angle. The base is 12 feet long, and the height is 5 feet. What length of plywood does Nate need for the ramp? *(Lesson 7-2)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression if $w = 8$, $x = 4$, $y = 2$, and $z = 5$.**

*(To review **evaluating expressions**, see page 736.)*

50. $\frac{1}{2}(7y)$

51. $\frac{1}{2}wx$

52. $\frac{1}{2}z(x + y)$

53. $\frac{1}{2}x(y + w)$

11-2 Areas of Triangles, Trapezoids, and Rhombi

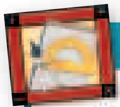
What You'll Learn

- Find areas of triangles.
- Find areas of trapezoids and rhombi.

How is the area of a triangle related to beach umbrellas?

Umbrellas can protect you from rain, wind, and sun. The umbrella shown at the right is made of triangular panels. To cover the umbrella frame with canvas panels, you need to know the area of each panel.

AREAS OF TRIANGLES You have learned how to find the areas of squares, rectangles, and parallelograms. The formula for the area of a triangle is related to these formulas.

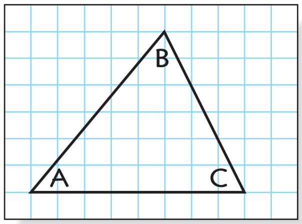


Geometry Activity

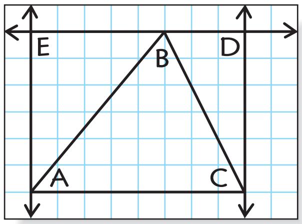
Area of a Triangle

Model

You can determine the area of a triangle by using the area of a rectangle.

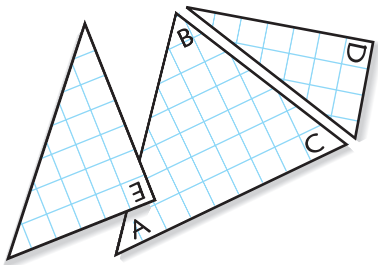


- Draw a triangle on grid paper so that one edge is along a horizontal line. Label the vertices on the interior of the angles of the triangle as *A*, *B*, and *C*.
- Draw a line perpendicular to $\overline{AC}$ through *A*.
- Draw a line perpendicular to $\overline{AC}$ through *C*.
- Draw a line parallel to $\overline{AC}$ through *B*.
- Label the points of intersection of the lines drawn as *D* and *E* as shown.
- Find the area of rectangle *ACDE* in square units.
- Cut out rectangle *ACDE*. Then cut out $\triangle ABC$. Place the two smaller pieces over $\triangle ABC$ to completely cover the triangle.



Analyze

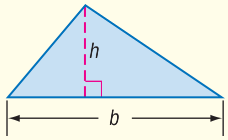
1. What do you observe about the two smaller triangles and $\triangle ABC$?
2. What fraction of rectangle *ACDE* is $\triangle ABC$?
3. Derive a formula that could be used to find the area of $\triangle ABC$.



The Geometry Activity suggests the formula for finding the area of a triangle.

Key Concept — *Area of a Triangle*

If a triangle has an area of A square units, a base of b units, and a corresponding height of h units, then $A = \frac{1}{2}bh$.



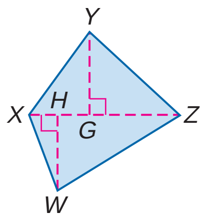
Study Tip

Look Back
To review **the height and altitude of a triangle**, see Lesson 5-1.

Example 1 Areas of Triangles

Find the area of quadrilateral *XYZW* if *XZ* = 39, *HW* = 20, and *YG* = 21.

The area of the quadrilateral is equal to the sum of the areas of $\triangle XWZ$ and $\triangle XYZ$.



area of $XYZW$ = area of $\triangle XYZ$ + area of $\triangle XWZ$

$= \frac{1}{2}bh_1 + \frac{1}{2}bh_2$

$= \frac{1}{2}(39)(21) + \frac{1}{2}(39)(20)$ Substitution

$= 409.5 + 390$ Simplify.

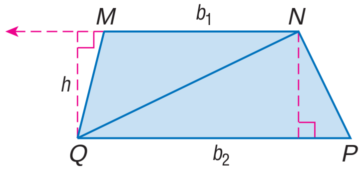
$= 799.5$

The area of quadrilateral $XYZW$ is 799.5 square units.

AREAS OF TRAPEZOIDS AND RHOMBI

The formulas for the areas of trapezoids and rhombi are related to the formula for the area of a triangle.

Trapezoid $MNPQ$ has diagonal $\overline{QN}$ with parallel bases $\overline{MN}$ and $\overline{PQ}$. Therefore, the altitude h from vertex Q to the extension of base $\overline{MN}$ is the same length as the altitude from vertex N to the base $\overline{QP}$. Since the area of the trapezoid is the area of two nonoverlapping parts, we can write the following equation.



area of trapezoid $MNPQ$ = area of $\triangle MNQ$ + area of $\triangle NPQ$

$A = \frac{1}{2}(b_1)h + \frac{1}{2}(b_2)h$ Let the area be A, MN be b_1, and QP be b_2.

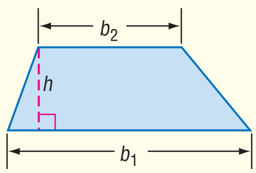
$A = \frac{1}{2}(b_1 + b_2)h$ Factor.

$A = \frac{1}{2}h(b_1 + b_2)$ Commutative Property

This is the formula for the area of any trapezoid.

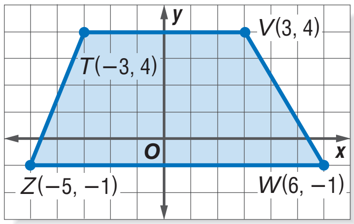
Key Concept — *Area of a Trapezoid*

If a trapezoid has an area of A square units, bases of b_1 units and b_2 units, and a height of h units, then $A = \frac{1}{2}h(b_1 + b_2)$.



Example 2 Area of a Trapezoid on the Coordinate Plane

COORDINATE GEOMETRY **Find the area of trapezoid *TVWZ* with vertices *T*(−3, 4), *V*(3, 4), *W*(6, −1), and *Z*(−5, −1).**



Bases: Since $\overline{TV}$ and $\overline{ZW}$ are horizontal, find their length by subtracting the *x*-coordinates of their endpoints.

$TV = |-3 - 3| = |-6|$ or 6 $\quad ZW = |-5 - 6| = |-11|$ or 11

Height: Because the bases are horizontal segments, the distance between them can be measured on a vertical line. That is, subtract the *y*-coordinates.

$h = |4 - (-1)|$ or 5

Area:

$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

$= \frac{1}{2}(5)(6 + 11)$ $h = 5, b_1 = 6, b_2 = 11$

$= 42.5$ Simplify.

The area of trapezoid *TVWZ* is 42.5 square units.

The formula for the area of a triangle can also be used to derive the formula for the area of a rhombus.

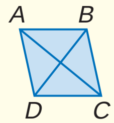
Study Tip

Area of a Rhombus Because a rhombus is also a parallelogram, you can also use the formula $A = bh$ to determine the area.

Key Concept — Area of a Rhombus

If a rhombus has an area of *A* square units and diagonals of d_1 and d_2 units, then $A = \frac{1}{2}d_1d_2$.

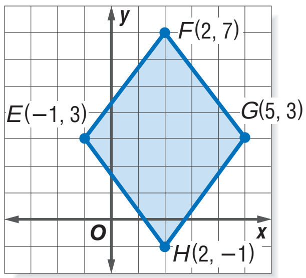
Example: $A = \frac{1}{2}(AC)(BD)$



You will derive this formula in Exercise 46.

Example 3 Area of a Rhombus on the Coordinate Plane

COORDINATE GEOMETRY **Find the area of rhombus *EFGH* with vertices at *E*(−1, 3), *F*(2, 7), *G*(5, 3), and *H*(2, −1).**



Explore To find the area of the rhombus, we need to know the lengths of each diagonal.

Plan Use coordinate geometry to find the length of each diagonal. Use the formula to find the area of rhombus *EFGH*.

Solve Let $\overline{EG}$ be d_1 and $\overline{FH}$ be d_2.

Subtract the *x*-coordinates of *E* and *G* to find that d_1 is 6.
Subtract the *y*-coordinates of *F* and *H* to find that d_2 is 8.

$A = \frac{1}{2}d_1d_2$ Area of a rhombus

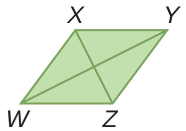
$= \frac{1}{2}(6)(8)$ or 24 $\quad d_1 = 6, d_2 = 8$

Examine The area of rhombus *EFGH* is 24 square units.

If you know all but one measure in a quadrilateral, you can solve for the missing measure using the appropriate area formula.

Example 4 Algebra: Find Missing Measures

a. Rhombus *WXYZ* has an area of 100 square meters. Find *WY* if *XZ* = 10 meters.



Use the formula for the area of a rhombus and solve for d_2.

$A = \frac{1}{2}d_1d_2$

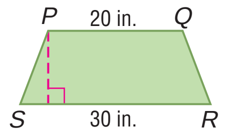
$100 = \frac{1}{2}(10)(d_2)$

$100 = 5d_2$

$20 = d_2$

WY is 20 meters long.

b. Trapezoid *PQRS* has an area of 250 square inches. Find the height of *PQRS*.



Use the formula for the area of a trapezoid and solve for h.

$A = \frac{1}{2}h(b_1 + b_2)$

$250 = \frac{1}{2}h(20 + 30)$

$250 = \frac{1}{2}(50)h$

$250 = 25h$

$10 = h$

The height of trapezoid *PQRS* is 10 inches.

Since the dimensions of congruent figures are equal, the areas of congruent figures are also equal.

Postulate 11.1

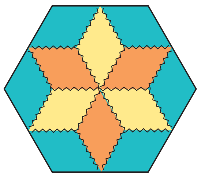
Congruent figures have equal areas.

Study Tip

Look Back
To review **the properties of rhombi and trapezoids,** see Lessons 8-5 and 8-6.

Example 5 Area of Congruent Figures

QUILTING **This quilt block is composed of twelve congruent rhombi arranged in a regular hexagon. The height of the hexagon is 8 inches. If the total area of the rhombi is 48 square inches, find the lengths of each diagonal and the area of one rhombus.**



First, find the area of one rhombus. From Postulate 11.1, the area of each rhombus is the same. So, the area of each rhombus is 48 ÷ 12 or 4 square inches.

Next, find the length of one diagonal. The height of the hexagon is equal to the sum of the long diagonals of two rhombi. Since the rhombi are congruent, the long diagonals must be congruent. So, the long diagonal is equal to 8 ÷ 2, or 4 inches.

Use the area formula to find the length of the other diagonal.

$A = \frac{1}{2}d_1d_2$ Area of a rhombus

$4 = \frac{1}{2}(4)\,d_2$ $A = 4, d_1 = 4$

$2 = d_2$ Solve for d_2.

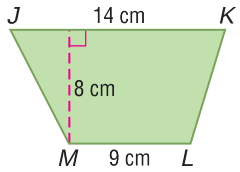
Each rhombus in the pattern has an area of 4 square inches and diagonals 4 inches and 2 inches long.

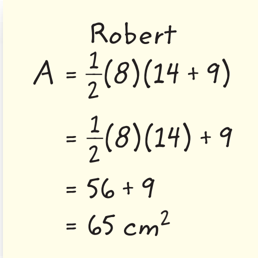
Check for Understanding

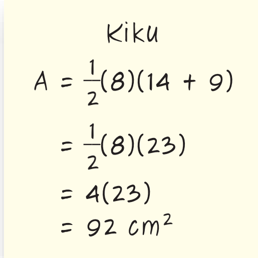
Concept Check

1. OPEN ENDED Draw an isosceles trapezoid that contains at least one isosceles triangle.

2. FIND THE ERROR Robert and Kiku are finding the area of trapezoid *JKLM*.







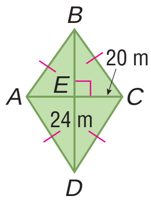
Who is correct? Explain your reasoning.

3. Determine whether it is *always, sometimes,* or *never* true that rhombi with the same area have the same diagonal lengths. Explain your reasoning.

Guided Practice

Find the area of each quadrilateral.

4.



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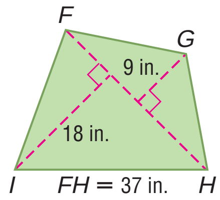

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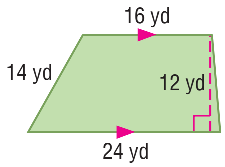


COORDINATE GEOMETRY **Find the area of each figure given the coordinates of the vertices.**

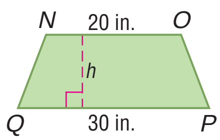
7. $\triangle ABC$ with $A(2, -3)$, $B(-5, -3)$, and $C(-1, 3)$

8. trapezoid *FGHJ* with $F(-1, 8)$, $G(5, 8)$, $H(3, 4)$, and $J(1, 4)$

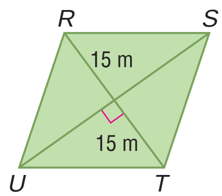
9. rhombus *LMPQ* with $L(-4, 3)$, $M(-2, 4)$, $P(0, 3)$, and $Q(-2, 2)$

ALGEBRA **Find the missing measure for each quadrilateral.**

10. Trapezoid *NOPQ* has an area of 250 square inches. Find the height of *NOPQ*.

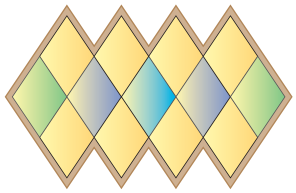


11. Rhombus *RSTU* has an area of 675 square meters. Find *SU*.



Application

12. INTERIOR DESIGN Jacques is designing a window hanging composed of 13 congruent rhombi. The total width of the window hanging is 15 inches, and the total area is $82\frac{7}{8}$ square inches. Find the length of each diagonal and the area of one rhombus.



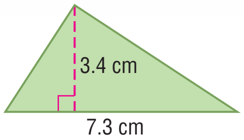
Practice and Apply

Homework Help

For Exercises	See Examples
13, 14, 19–21	1
15, 16, 22–25	2
17, 18, 26–29	3
30–35, 40–44	4
36–39	5

Extra Practice
See page 776.

Find the area of each figure. Round to the nearest tenth if necessary.

13.


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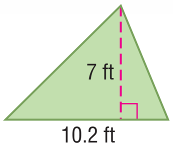

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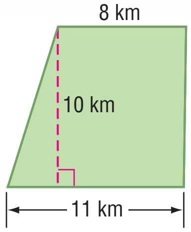

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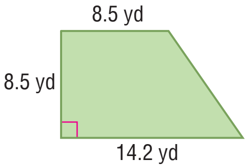

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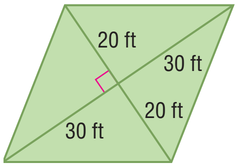

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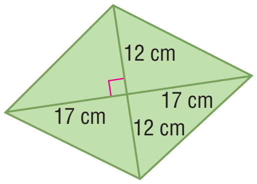

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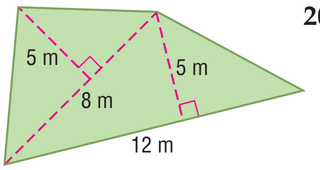

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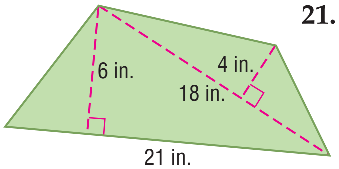

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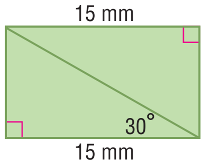


COORDINATE GEOMETRY **Find the area of trapezoid *PQRT* given the coordinates of the vertices.**

22. $P(0, 3)$, $Q(3, 7)$, $R(5, 7)$, $T(6, 3)$

23. $P(-4, -5)$, $Q(-2, -5)$, $R(4, 6)$, $T(-4, 6)$

24. $P(-3, 8)$, $Q(6, 8)$, $R(6, 2)$, $T(1, 2)$

25. $P(-6, 3)$, $Q(1, 3)$, $R(-2, -2)$, $T(-4, -2)$

COORDINATE GEOMETRY **Find the area of rhombus *JKLM* given the coordinates of the vertices.**

26. $J(2, 1)$, $K(7, 4)$, $L(12, 1)$, $M(7, -2)$

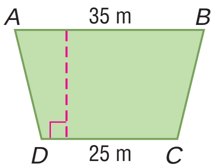
27. $J(-1, 2)$, $K(1, 7)$, $L(3, 2)$, $M(1, -3)$

28. $J(-1, -4)$, $K(2, 2)$, $L(5, -4)$, $M(2, -10)$

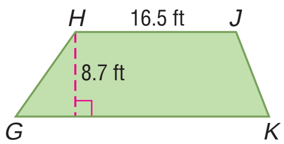
29. $J(2, 4)$, $K(6, 6)$, $L(10, 4)$, $M(6, 2)$

ALGEBRA **Find the missing measure for each figure.**

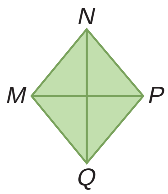
30. Trapezoid *ABCD* has an area of 750 square meters. Find the height of *ABCD*.



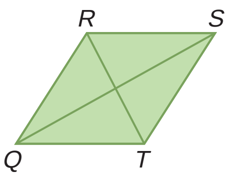
31. Trapezoid *GHJK* has an area of 188.35 square feet. If *HJ* is 16.5 feet, find *GK*.



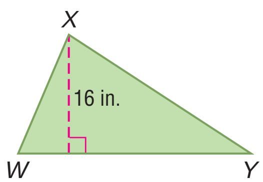
32. Rhombus *MNPQ* has an area of 375 square inches. If *MP* is 25 inches, find *NQ*.



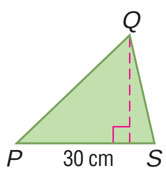
33. Rhombus *QRST* has an area of 137.9 square meters. If *RT* is 12.2 meters, find *QS*.



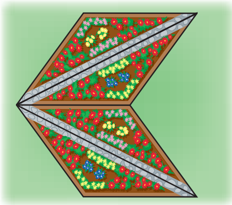
34. Triangle *WXY* has an area of 248 square inches. Find the length of the base.



35. Triangle *PQS* has an area of 300 square centimeters. Find the height.



GARDENS For Exercises 36 and 37, use the following information.



Keisha designed a garden that is shaped like two congruent rhombi. She wants the long diagonals lined with a stone walkway. The total area of the garden is 150 square feet, and the shorter diagonals are each 12 feet long.

36. Find the length of each stone walkway.

37. Find the length of each side of the garden.

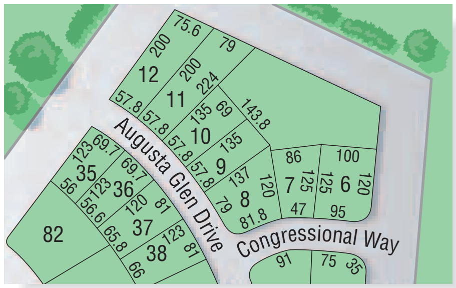
Career Choices

Real Estate Agent

Real estate agents must have a broad knowledge of the neighborhoods in their community. About two-thirds of real estate agents are self-employed. Success is dependent on selling properties.

Source: ww.bls.gov

REAL ESTATE For Exercises 38 and 39, use the following information.



The map shows the layout and dimensions of several lot parcels in Linworth Village. Suppose Lots 35 and 12 are trapezoids.

38. If the height of Lot 35 is 122.81 feet, find the area of this lot.

39. If the height of Lot 12 is 199.8 feet, find the area of this lot.

Online Research **Data Update** Use the Internet or other resource to find the median price of homes in the United States. How does this compare to the median price of homes in your community? Visit www.geometryonline.com/data_update to learn more.

Find the area of each figure.

40. rhombus with a perimeter of 20 meters and a diagonal of 8 meters

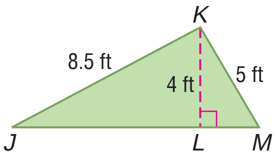
41. rhombus with a perimeter of 52 inches and a diagonal of 24 inches

42. isosceles trapezoid with a perimeter of 52 yards; the measure of one base is 10 yards greater than the other base, the measure of each leg is 3 less than twice the length of the shorter base

43. equilateral triangle with a perimeter of 15 inches

44. scalene triangle with sides that measure 34.0 meters, 81.6 meters, and 88.4 meters.

45. Find the area of $\triangle JKM$.

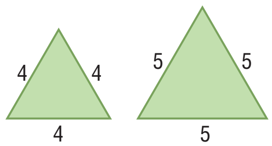


46. Derive the formula for the area of a rhombus using the formula for the area of a triangle.

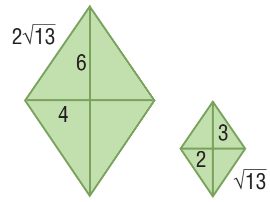
47. Determine whether the statement *Two triangles that have the same area also have the same perimeter* is true or false. Give an example or counterexample.

Each pair of figures is similar. Find the area and perimeter of each figure. Describe how changing the dimensions affects the perimeter and area.

48.



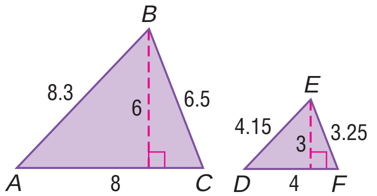
49.



50. **RECREATION** Becky wants to cover a kite frame with decorative paper. If the length of one diagonal is 20 inches and the other diagonal measures 25 inches, find the area of the surface of the kite.

SIMILAR FIGURES **For Exercises 51–56, use the following information.**

Triangle *ABC* is similar to triangle *DEF*.



51. Find the scale factor.

52. Find the perimeter of each triangle.

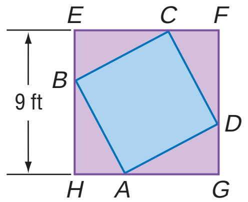
53. Compare the ratio of the perimeters of the triangles to the scale factor.

54. Find the area of each triangle.

55. Compare the ratio of the areas of the triangles to the scale factor.

56. Compare the ratio of the areas of the triangles to the ratio of the perimeters of the triangles.

57. **CRITICAL THINKING** In the figure, the vertices of quadrilateral *ABCD* intersect square *EFGH* and divide its sides into segments with measures that have a ratio of 1:2. Find the area of *ABCD*. Describe the relationship between the areas of *ABCD* and *EFGH*.



58. *WRITING IN MATH* Answer the question that was posed at the beginning of the lesson.

How is the area of a triangle related to beach umbrellas?

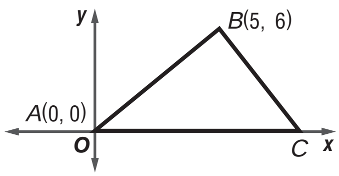
Include the following in your answer:
- how to find the area of a triangle, and
- how the area of a triangle can help you find the areas of rhombi and trapezoids.

Standardized Test Practice

59. In the figure, if point *B* lies on the perpendicular bisector of $\overline{AC}$, what is the area of $\triangle ABC$?

Ⓐ 15 units^2 Ⓑ 30 units^2

Ⓒ 50 units^2 Ⓓ 1602 units^2



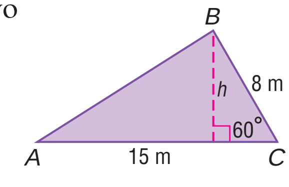
60. **ALGEBRA** What are the solutions of the equation $(2x - 7)(x + 10) = 0$?

Ⓐ -3.5 and 10 Ⓑ 7 and -10 Ⓒ $\frac{2}{7}$ and -10 Ⓓ 3.5 and -10

Extending the Lesson

Trigonometric Ratios and the Areas of Triangles

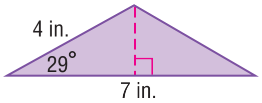
The area of any triangle can be found given the measures of two sides of the triangle and the measure of the included angle. Suppose we are given $AC = 15$, $BC = 8$, and $m\angle C = 60$. To find the height of the triangle, use the sine ratio, $\sin C = \frac{h}{BC}$. Then use the value of *h* in the formula for the area of a triangle. So, the area is $\frac{1}{2}(15)(8 \sin 60°)$ or 52.0 square meters.



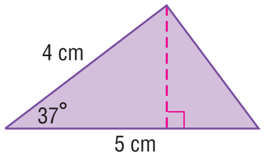
61. Derive a formula to find the area of any triangle, given the measures of two sides of the triangle and their included angle.

Find the area of each triangle. Round to the nearest hundredth.

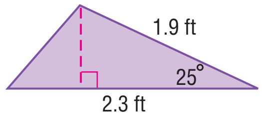
62.



63.



64.

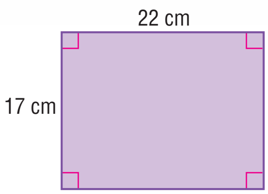


Maintain Your Skills

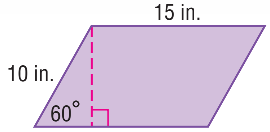
Mixed Review

Find the area of each figure. Round to the nearest tenth. *(Lesson 11-1)*

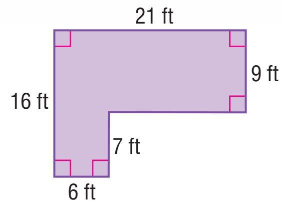
65.



66.



67.



Write an equation of circle R based on the given information. *(Lesson 10-8)*

68. center: $R(1, 2)$
radius: 7

69. center: $R\left(-4, \frac{1}{2}\right)$
radius: $\frac{11}{2}$

70. center: $R(-1.3, 5.6)$
radius: 3.5

71. CRAFTS Andria created a pattern to appliqué flowers onto a quilt by first drawing a regular pentagon that was 3.5 inches long on each side. Then she added a semicircle onto each side of the pentagon to create the appearance of five petals. How many inches of gold trim does she need to edge 10 flowers? *(Lesson 10-1)*

Given the magnitude and direction of a vector, find the component form with values rounded to the nearest tenth. *(Lesson 9-6)*

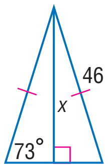
72. magnitude of 136 at a direction of 25 degrees with the positive x-axis

73. magnitude of 280 at a direction of 52 degrees with the positive x-axis

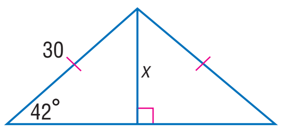
Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find x. Round to the nearest tenth.**
*(To review **trigonometric ratios in right triangles**, see Lesson 7-4.)*

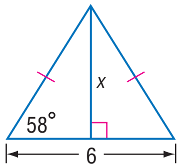
74.



75.



76.



Practice Quiz 1

Lessons 11-1 and 11-2

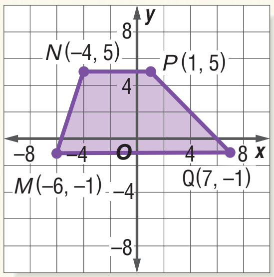
The coordinates of the vertices of quadrilateral $JKLM$ are $J(-8, 4)$, $K(-4, 0)$, $L(0, 4)$, and $M(-4, 8)$. *(Lesson 11-1)*

1. Determine whether $JKLM$ is a *square*, a *rectangle*, or a *parallelogram*.

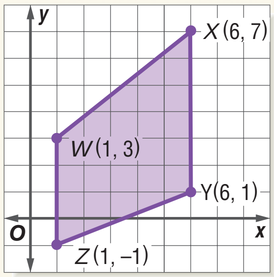
2. Find the area of $JKLM$.

Find the area of each trapezoid. *(Lesson 11-2)*

3.



4.



5. The area of a rhombus is 546 square yards. If d_1 is 26 yards long, find the length of d_2. *(Lesson 11-2)*

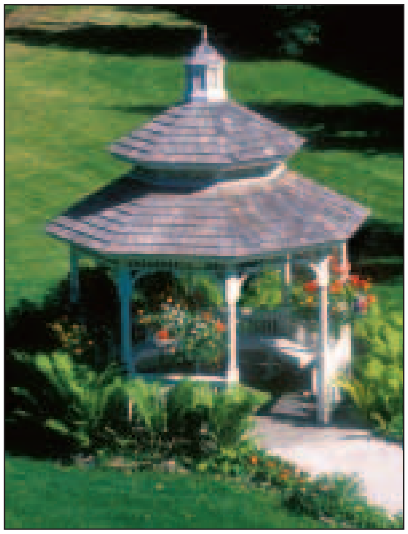
11-3 Areas of Regular Polygons and Circles

What You'll Learn

- Find areas of regular polygons.
- Find areas of circles.

How can you find the area of a polygon?

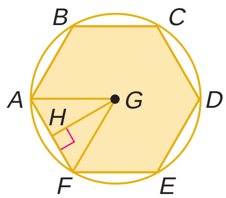
The foundations of most gazebos are shaped like regular hexagons. Suppose the owners of this gazebo would like to install tile on the floor. If tiles are sold in square feet, how can they find out the actual area of tiles needed to cover the floor?



Vocabulary

- apothem

AREAS OF REGULAR POLYGONS In regular hexagon *ABCDEF* inscribed in circle *G*, $\overline{GA}$ and $\overline{GF}$ are radii from the center of the circle *G* to two vertices of the hexagon. $\overline{GH}$ is drawn from the center of the regular polygon perpendicular to a side of the polygon. This segment is called an **apothem**.

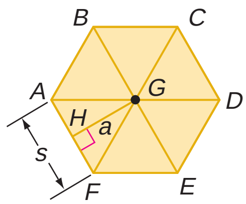


Study Tip

Look Back
To review **apothem**, see Lesson 1-4.

Triangle *GFA* is an isosceles triangle, since the radii are congruent. If all of the radii were drawn, they would separate the hexagon into 6 nonoverlapping congruent isosceles triangles.

The area of the hexagon can be determined by adding the areas of the triangles. Since $\overline{GH}$ is perpendicular to $\overline{AF}$, it is an altitude of $\triangle AGF$. Let *a* represent the length of $\overline{GH}$ and let *s* represent the length of a side of the hexagon.



$$\text{Area of } \triangle AGF = \frac{1}{2}bh$$
$$= \frac{1}{2}sa$$

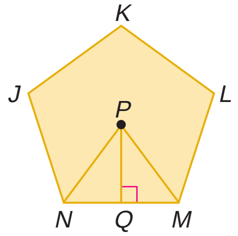
The area of one triangle is $\frac{1}{2}sa$ square units. So the area of the hexagon is $6\left(\frac{1}{2}sa\right)$ square units. Notice that the perimeter *P* of the hexagon is 6*s* units. We can substitute *P* for 6*s* in the area formula. So, $A = 6\left(\frac{1}{2}sa\right)$ becomes $A = \frac{1}{2}Pa$. This formula can be used for the area of any regular polygon.

Key Concept — *Area of a Regular Polygon*

If a regular polygon has an area of *A* square units, a perimeter of *P* units, and an apothem of *a* units, then $A = \frac{1}{2}Pa$.

Example 1 Area of a Regular Polygon

Find the area of a regular pentagon with a perimeter of 40 centimeters.



Apothem: The central angles of a regular pentagon are all congruent. Therefore, the measure of each angle is $\frac{360}{5}$ or 72. $\overline{PQ}$ is an apothem of pentagon $JKLMN$. It bisects $\angle NPM$ and is a perpendicular bisector of $\overline{NM}$. So, $m\angle MPQ = \frac{1}{2}(72)$ or 36. Since the perimeter is 40 centimeters, each side is 8 centimeters and $QM = 4$ centimeters. Write a trigonometric ratio to find the length of $\overline{PQ}$.

$$\tan \angle MPQ = \frac{QM}{PQ} \qquad \tan \theta = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

$$\tan 36° = \frac{4}{PQ} \qquad m\angle MPQ = 36,\ QM = 4$$

$$(PQ) \tan 36° = 4 \qquad \text{Multiply each side by } PQ.$$

$$PQ = \frac{4}{\tan 36°} \qquad \text{Divide each side by } \tan 36°.$$

$$PQ \approx 5.5 \qquad \text{Use a calculator.}$$

Area:

$$A = \frac{1}{2}Pa \qquad \text{Area of a regular polygon}$$

$$\approx \frac{1}{2}(40)(5.5) \qquad P = 40,\ a \approx 5.5$$

$$\approx 110 \qquad \text{Simplify.}$$

So, the area of the pentagon is about 110 square centimeters.

Study Tip

Problem Solving
There is another method for finding the apothem of a regular polygon. You can use the Interior Angle Sum Theorem to find $m\angle PMQ$ and then write a trigonometric ratio to find PQ.

AREAS OF CIRCLES You can use a calculator to help derive the formula for the area of a circle from the areas of regular polygons.

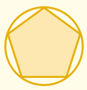
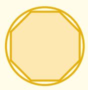
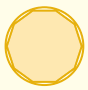
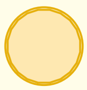
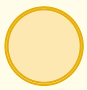
Geometry Activity

Area of a Circle

Collect Data

Suppose each regular polygon is inscribed in a circle of radius r.

1. Copy and complete the following table. Round to the nearest hundredth.

Inscribed Polygon						
Number of Sides	3	5	8	10	20	50
Measure of a Side	1.73r	1.18r	0.77r	0.62r	0.31r	0.126r
Measure of Apothem	0.5r	0.81r	0.92r	0.95r	0.99r	0.998r
Area						

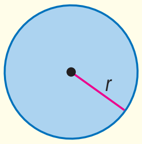
Analyze the Data

2. What happens to the appearance of the polygon as the number of sides increases?
3. What happens to the areas as the number of sides increases?
4. **Make a conjecture** about the formula for the area of a circle.

You can see from the Geometry Activity that the more sides a regular polygon has, the more closely it resembles a circle.

Key Concept — *Area of a Circle*

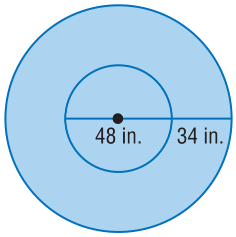
If a circle has an area of A square units and a radius of r units, then $A = \pi r^2$.



> **Study Tip**
>
> *Square Yards*
> A square yard measures 36 inches by 36 inches or 1296 square inches.

Example 2 Use Area of a Circle to Solve a Real-World Problem

SEWING **A caterer has a 48-inch diameter table that is 34 inches tall. She wants a tablecloth that will touch the floor. Find the area of the tablecloth in square yards.**



The diameter of the table is 48 inches, and the tablecloth must extend 34 inches in each direction. So the diameter of the tablecloth is 34 + 48 + 34 or 116 inches. Divide by 2 to find that the radius is 58 inches.

$A = \pi r^2$	Area of a circle
$= \pi(58)^2$	Substitution
$\approx 10{,}568.3$	Use a calculator.

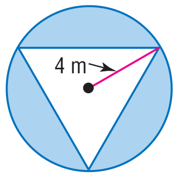
The area of the tablecloth is 10,568.3 square inches. To convert to square yards, divide by 1296. The area of the tablecloth is 8.2 square yards to the nearest tenth.

> **Study Tip**
>
> *Look Back*
> To review **inscribed and circumscribed polygons,** see Lesson 10-4.

You can use the properties of circles and regular polygons to find the areas of inscribed and circumscribed polygons.

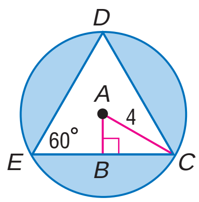
Example 3 Area of an Inscribed Polygon

Find the area of the shaded region. Assume that the triangle is equilateral.

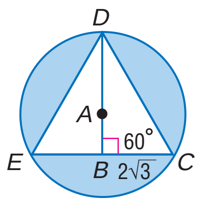


The area of the shaded region is the difference between the area of the circle and the area of the triangle. First, find the area of the circle.

$A = \pi r^2$	Area of a circle
$= \pi(4)^2$	Substitution
≈ 50.3	Use a calculator.



To find the area of the triangle, use properties of 30°-60°-90° triangles. First, find the length of the base. The hypotenuse of $\triangle ABC$ is 4, so BC is $2\sqrt{3}$. Since $EC = 2(BC)$, $EC = 4\sqrt{3}$.



Next, find the height of the triangle, DB. Since $m\angle DCB$ is 60, $DB = 2\sqrt{3}(\sqrt{3})$ or 6.

Use the formula to find the area of the triangle.

$A = \frac{1}{2}bh$	Area of a triangle
$= \frac{1}{2}(4\sqrt{3})(6)$	$b = 4\sqrt{3}$, $h = 6$
≈ 20.8	Use a calculator.

The area of the shaded region is 50.3 − 20.8 or 29.5 square meters to the nearest tenth.

Check for Understanding

Concept Check

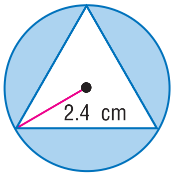
1. **Explain** how to derive the formula for the area of a regular polygon.
2. **OPEN ENDED** Describe another method to find the base or height of a right triangle given one acute angle and the length of one side.

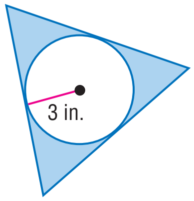
Guided Practice

Find the area of each polygon. Round to the nearest tenth.

3. a regular hexagon with a perimeter of 42 yards
4. a regular nonagon with a perimeter of 108 meters

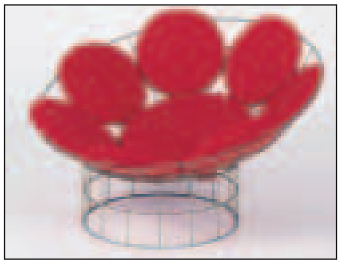
Find the area of each shaded region. Assume that all polygons that appear to be regular are regular. Round to the nearest tenth.

5. 

6. 

Application

7. **UPHOLSTERY** Tyra wants to cover the cushions of her papasan chair with a different fabric. If there are seven circular cushions that are the same size with a diameter of 12 inches, around a center cushion with a diameter of 20 inches, find the area of fabric in square yards that she will need to cover both sides of the cushions. Allow an extra 3 inches of fabric around each cushion.



Practice and Apply

Homework Help

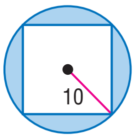
For Exercises	See Examples
8–13, 26, 27	1
14–23, 37–42	3
24, 25, 28–31	2

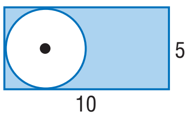
Extra Practice
See page 777.

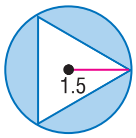
Find the area of each polygon. Round to the nearest tenth.

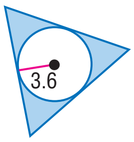
8. a regular octagon with a perimeter of 72 inches
9. a square with a perimeter of $84\sqrt{2}$ meters
10. a square with apothem length of 12 centimeters
11. a regular hexagon with apothem length of 24 inches
12. a regular triangle with side length of 15.5 inches
13. a regular octagon with side length of 10 kilometers

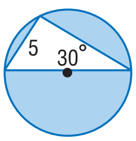
Find the area of each shaded region. Assume that all polygons that appear to be regular are regular. Round to the nearest tenth.

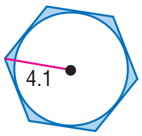
14. 

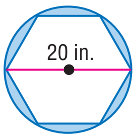
15. 

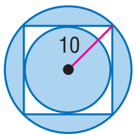
16. 

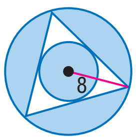
17. 

18. 

19. 

20. 

21. 

22. 

23. **ALGEBRA** A circle is inscribed in a square, which is circumscribed by another circle. If the diagonal of the square is $2x$, find the ratio of the area of the large circle to the area of the small circle.

24. **CAKE** A bakery sells single-layer mini-cakes that are 3 inches in diameter for $4 each. They also have a 9-inch cake for $15. If both cakes are the same thickness, which option gives you more cake for the money, nine mini-cakes or one 9-inch cake? Explain.

25. **PIZZA** A pizza shop sells 8-inch pizzas for $5 and 16-inch pizzas for $10. Which would give you more pizza, two 8-inch pizzas or one 16-inch pizza? Explain.

COORDINATE GEOMETRY **The coordinates of the vertices of a regular polygon are given. Find the area of each polygon to the nearest tenth.**

26. $T(0, 0), U(-7, -7), V(0, -14), W(7, -7)$

27. $G(-12, 0), H(0, 4\sqrt{3}), J(0, -4\sqrt{3})$

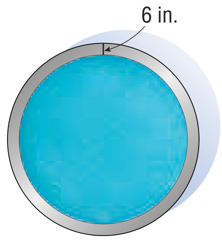
28. $J(5, 0), K(4, -4), L(0, -5), M(-4, -4), N(-5, 0), P(-4, 4), Q(0, 5), R(4, 4)$

29. $A(-3, 3), B(0, 4), C(3, 3), D(4, 0), E(3, -3), F(0, -4), G(-3, -3), H(-4, 0)$

Find the area of a circle having the given circumference. Round to the nearest tenth.

30. 34π 31. 17π 32. 54.8 33. 91.4

SWIMMING POOL **For Exercises 34 and 35, use the following information.**
The area of a circular pool is approximately 7850 square feet. The owner wants to replace the tiling at the edge of the pool.



34. The edging is 6 inches wide, so she plans to use 6-inch square tiles to form a continuous inner edge. How many tiles will she need to purchase?

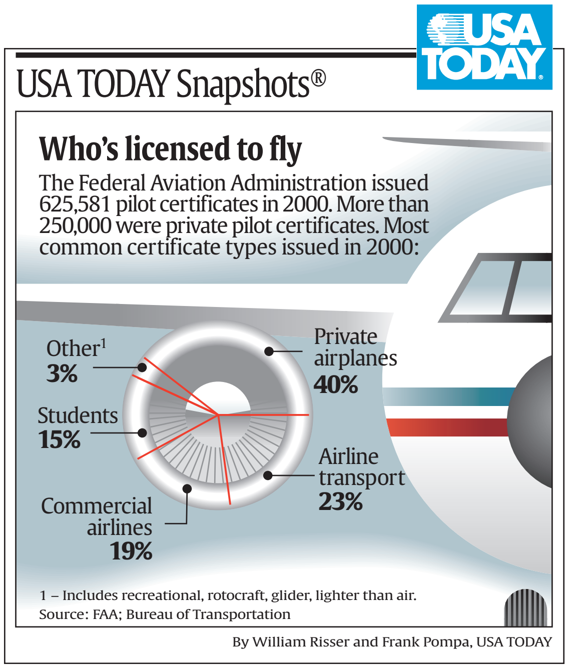
35. Once the square tiles are in place around the pool, there will be extra space between the tiles. What shape of tile will best fill this space? How many tiles of this shape should she purchase?

AVIATION **For Exercises 36–38, refer to the circle graph.**

36. Suppose the radius of the circle on the graph is 1.3 centimeters. Find the area of the circle on the graph.

37. Francesca wants to use this circle graph for a presentation. She wants the circle to use as much space on a 22″ by 28″ sheet of poster board as possible. Find the area of the circle.

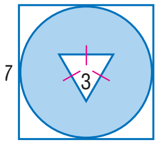
38. **CRITICAL THINKING** Make a conjecture about how you could determine the area of the region representing the pilots who are certified to fly private airplanes.

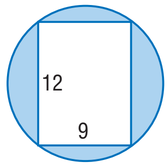


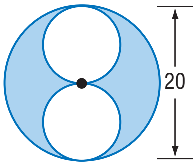
Study Tip

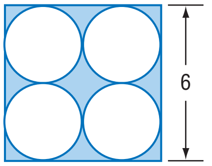
Look Back
To review **circle graphs,** see Lesson 10-2.

Find the area of each shaded region. Round to the nearest tenth.

39.


40.


41.


42.


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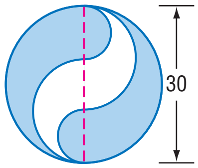

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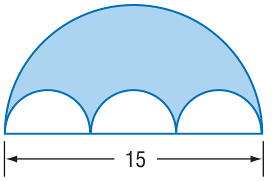


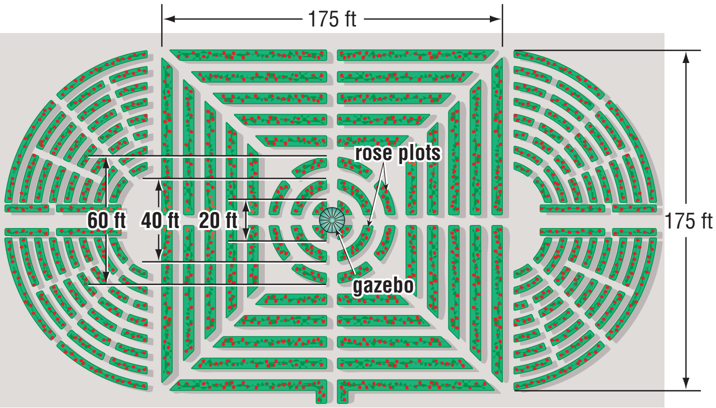
More About. . .

Architecture

This structure is a *pagoda*. Pagodas are characterized by having several hexagonal or octagonal stories each topped with a curved roof. In this temple, the sizes of the floors are in the ratio 1 : 3 : 5 : 7.

Source: www.infoplease.com

GARDENS **For Exercises 45–47, use the following information.**
The Elizabeth Park Rose Garden in Hartford, Connecticut, was designed with a gazebo surrounded by two concentric rose garden plots. Wide paths emanate from the center, dividing the garden into square and circular sections.



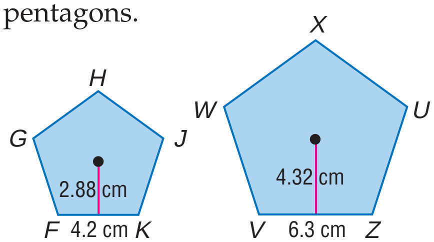
45. Find the area and perimeter of the entire Rose Garden. Round to the nearest tenth.

46. What is the total of the circumferences of the three concentric circles formed by the gazebo and the two circular rose garden plots? (Ignore the width of the rose plots and the width of the paths.)

47. Each rose plot has a width of 5 feet. What is the area of the path between the outer two complete circles of rose garden plots?

48. **ARCHITECTURE** The Anraku-ji Temple in Japan is composed of four octagonal floors of different sizes that are separated by four octagonal roofs of different sizes. Refer to the information at the left. Determine whether the areas of each of the four floors are in the same ratio as their sizes. Explain.

SIMILAR FIGURES **For Exercises 49–54, use the following information.**
Polygons *FGHJK* and *VWXUZ* are similar regular pentagons.



49. Find the scale factor.

50. Find the perimeter of each pentagon.

51. Compare the ratio of the perimeters of the pentagons to the scale factor.

52. Find the area of each pentagon.

53. Compare the ratio of the areas of the pentagons to the scale factor.

54. Compare the ratio of the areas of the pentagons to the ratio of the perimeters of the pentagons.

Sakamoto Photo Research Laboratory/CORBIS

55. **CRITICAL THINKING** A circle inscribes one regular hexagon and circumscribes another. If the radius of the circle is 10 units long, find the ratio of the area of the smaller hexagon to the area of the larger hexagon.

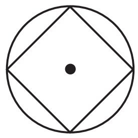
56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you find the area of a polygon?

Include the following in your answer:
- information needed about the gazebo floor to find the area, and
- how to find the number of tiles needed to cover the floor.

Standardized Test Practice

57. A square is inscribed in a circle of area 18π square units. Find the length of a side of the square.



Ⓐ 3 units Ⓑ 6 units
Ⓒ $3\sqrt{2}$ units Ⓓ $6\sqrt{2}$ units

58. **ALGEBRA** The average of x numbers is 15. If the sum of the x numbers is 90, what is the value of x?

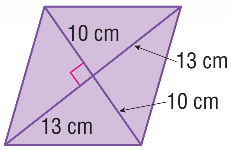
Ⓐ 5 Ⓑ 6 Ⓒ 8 Ⓓ 15

Maintain Your Skills

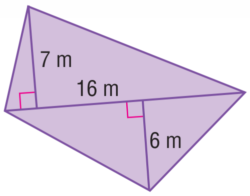
Mixed Review

Find the area of each quadrilateral. *(Lesson 11-2)*

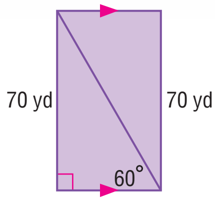
59.



60.



61.



COORDINATE GEOMETRY **Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral.** *(Lesson 11-1)*

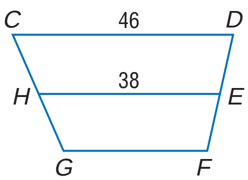
62. $A(-3, 2), B(4, 2), C(2, -1), D(-5, -1)$

63. $F(4, 1), G(4, -5), H(-2, -5), J(-2, 1)$

64. $K(-1, -3), L(-2, 5), M(1, 5), N(2, -3)$

65. $P(5, -7), Q(-1, -7), R(-1, -2), S(5, -2)$

Refer to trapezoid *CDFG* with median $\overline{HE}$.
(Lesson 8-6)



66. Find *GF*.

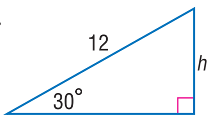
67. Let $\overline{WX}$ be the median of *CDEH*. Find *WX*.

68. Let $\overline{YZ}$ be the median of *HEFG*. Find *YZ*.

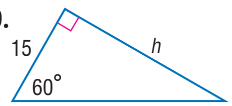
Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find *h*.** *(To review **special right triangles**, see Lesson 7-3.)*

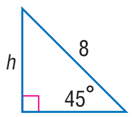
69.



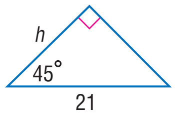
70.



71.



72.



11-4 Areas of Irregular Figures

What You'll Learn

- Find areas of irregular figures.
- Find areas of irregular figures on the coordinate plane.

How do windsurfers use area?

The sail for a windsurf board cannot be classified as a triangle or a parallelogram. However, it can be separated into figures that can be identified, such as trapezoids and a triangle.

Vocabulary

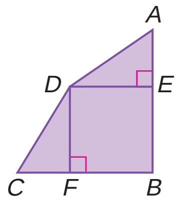
- irregular figure
- irregular polygon

Study Tip

Reading Math
Irregular figures are also called *composite figures* because the regions can be separated into smaller regions.

IRREGULAR FIGURES An **irregular figure** is a figure that cannot be classified into the specific shapes that we have studied. To find areas of irregular figures, separate the figure into shapes of which we can find the area. Draw auxiliary lines as necessary. The sum of the areas of each is the area of the figure.

Auxiliary lines are drawn in quadrilateral *ABCD*. $\overline{DE}$ and $\overline{DF}$ separate the figure into $\triangle ADE$, $\triangle CDF$, and rectangle *BEDF*.

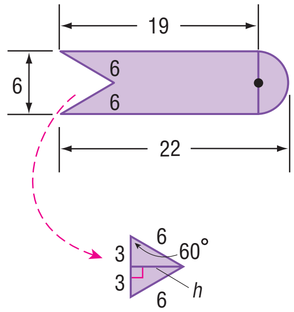


Postulate 11.2

The area of a region is the sum of all of its nonoverlapping parts.

Example 1 Area of an Irregular Figure

Find the area of the figure.



The figure can be separated into a rectangle with dimensions 6 units by 19 units, an equilateral triangle with sides each measuring 6 units, and a semicircle with a radius of 3 units.

Use 30°-60°-90° relationships to find that the height of the triangle is $3\sqrt{3}$.

area of irregular figure = area of rectangle − area of triangle + area of semicircle

$$= \ell w - \frac{1}{2}bh + \frac{1}{2}\pi r^2 \quad \text{Area formulas}$$

$$= 19 \cdot 6 - \frac{1}{2}(6)(3\sqrt{3}) + \frac{1}{2}\pi(3^2) \quad \text{Substitution}$$

$$= 114 - 9\sqrt{3} + \frac{9}{2}\pi \quad \text{Simplify.}$$

$$\approx 112.5 \quad \text{Use a calculator.}$$

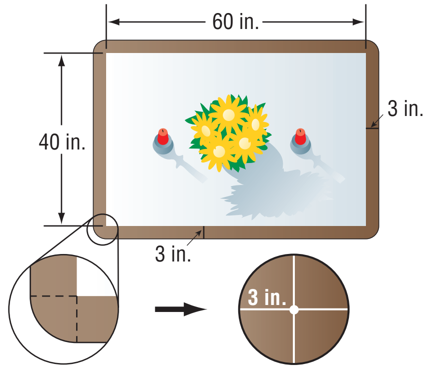
The area of the irregular figure is 112.5 square units to the nearest tenth.

www.geometryonline.com/extra_examples

CONTENTS

Example 2 Find the Area of an Irregular Figure to Solve a Problem

FURNITURE **Melissa's dining room table has hardwood around the outside. Find the area of wood around the edge of the table.**



First, draw auxiliary lines to separate the figure into regions. The table can be separated into four rectangles and four corners.

The four corners of the table form a circle with radius 3 inches.

area of wood edge = area of rectangles + area of circle

$= 2\ell w + 2\ell w + \pi r^2$ Area formulas

$= 2(3)(60) + 2(3)(40) + \pi(3^2)$ Substitution

$= 360 + 240 + 9\pi$ Simplify.

≈ 628.3 Use a calculator.

The area of the wood edge of the table is 628.3 square inches to the nearest tenth.

WebQuest

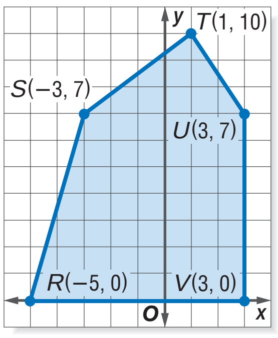
Identifying the polygons forming a region such as a tessellation will help you determine the type of tessellation. Visit www.geometryonline.com/webquest to continue work on your WebQuest project.

IRREGULAR FIGURES ON THE COORDINATE PLANE

The formula for the area of a regular polygon does not apply to an **irregular polygon**, a polygon that is not regular. To find the area of an irregular polygon on the coordinate plane, separate the polygon into known figures.

Example 3 Coordinate Plane

COORDINATE GEOMETRY **Find the area of polygon *RSTUV*.**



First, separate the figure into regions. Draw an auxiliary line from *S* to *U*. This divides the figure into triangle *STU* and trapezoid *RSUV*.

Find the difference between *x*-coordinates to find the length of the base of the triangle and the lengths of the bases of the trapezoid. Find the difference between *y*-coordinates to find the heights of the triangle and trapezoid.

area of *RSTUV* = area of $\triangle STU$ + area of trapezoid *RSUV*

$= \frac{1}{2}bh + \frac{1}{2}h(b_1 + b_2)$ Area formulas

$= \frac{1}{2}(6)(3) + \frac{1}{2}(7)(8 + 6)$ Substitution

$= 58$ Simplify.

The area of *RSTUV* is 58 square units.

Study Tip

Estimation

Estimate the area of the simple closed curves by counting the unit squares. Use the estimate to determine if your answer is reasonable.

Check for Understanding

Concept Check

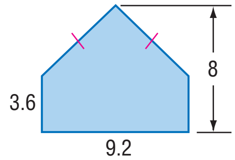
1. **OPEN ENDED** Sketch an irregular figure on a coordinate plane and find its area.

2. **Describe** the difference between an irregular figure and an irregular polygon.

Guided Practice

Find the area of each figure. Round to the nearest tenth if necessary.

3.



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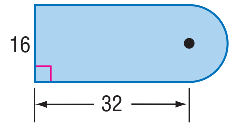
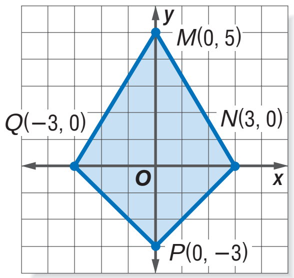


COORDINATE GEOMETRY **Find the area of each figure.**

5.



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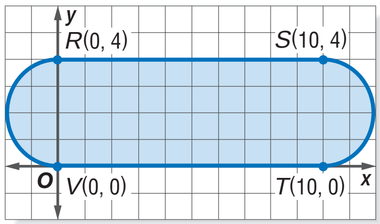
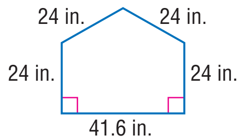


Application

7. **GATES** The Roths have a series of interlocking gates to form a play area for their baby. Find the area enclosed by the wall and gates.



Practice and Apply

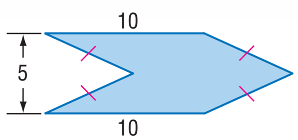
Homework Help

For Exercises	See Examples
8–13	1
14, 15, 23–27	2
16–22	3

Extra Practice
See page 777.

Find the area of each figure. Round to the nearest tenth if necessary.

8.



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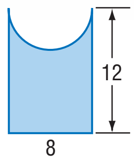

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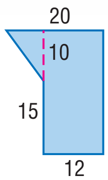

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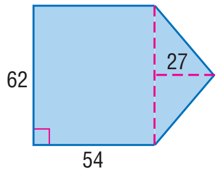
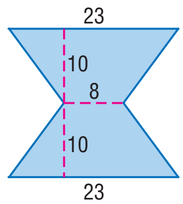
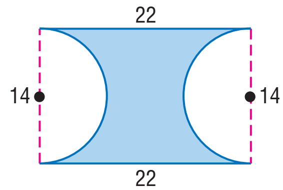


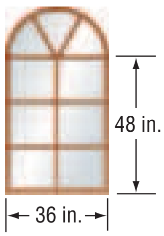
12.



13.



WINDOWS **For Exercises 14 and 15, use the following information.**
Mr. Cortez needs to replace this window in his house. The window panes are rectangles and sectors.



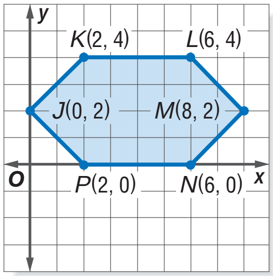
14. Find the perimeter of the window.

15. Find the area of the window.

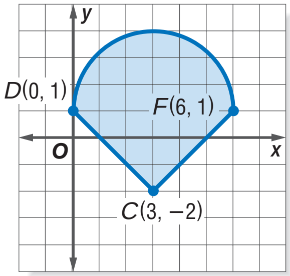
CONTENTS

COORDINATE GEOMETRY **Find the area of each figure. Round to the nearest tenth if necessary.**

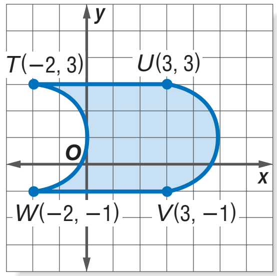
16.



17.



18.



COORDINATE GEOMETRY **The vertices of an irregular figure are given. Find the area of each figure.**

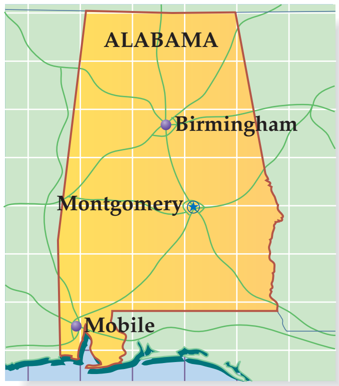
19. $M(-4, 0)$, $N(0, 3)$, $P(5, 3)$, $Q(5, 0)$

20. $T(-4, -2)$, $U(-2, 2)$, $V(3, 4)$, $W(3, -2)$

21. $G(-3, -1)$, $H(-3, 1)$, $I(2, 4)$, $J(5, -1)$, $K(1, -3)$

22. $P(-8, 7)$, $Q(3, 7)$, $R(3, -2)$, $S(-1, 3)$, $T(-11, 1)$

23. **GEOGRAPHY** Estimate the area of the state of Alabama. Each square on the grid represents 2500 square miles.



24. **RESEARCH** Find a map of your state or a state of your choice. Estimate the area. Then use the Internet or other source to check the accuracy of your estimate.

More About. . .

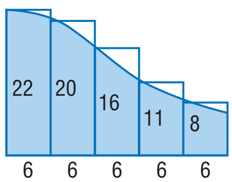
Calculus

Twenty percent of students in Japan take a calculus class in high school. In Korea and Switzerland, nearly all high school students take a course in calculus. In the U.S., only 1% of students take a calculus course in high school.

Source: NCTM

CALCULUS **For Exercises 25–27, use the following information.**

The irregular region under the curve has been separated into rectangles of equal width.

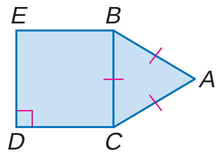


25. Use the rectangles to approximate the area of the region.

26. Analyze the estimate. Do you think the actual area is larger or smaller than your estimate? Explain.

27. How could the irregular region be separated to give an estimate of the area that is more accurate?

28. **CRITICAL THINKING** Find the ratio of the area of $\triangle ABC$ to the area of square $BCDE$.



29. ***WRITING IN MATH*** Answer the question that was posed at the beginning of the lesson.

How do windsurfers use area?

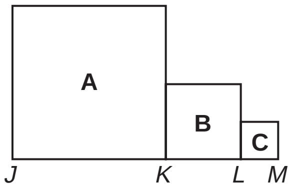
Include the following in your answer:

- describe how to find the area of the sail, and
- another example of an irregular figure.

Mark S. Wexler/Woodfin Camp & Associates

Standardized Test Practice

30. In the figure consisting of squares A, B, and C, $JK = 2KL$ and $KL = 2LM$. If the perimeter of the figure is 66 units, what is the area?



Ⓐ 117 units^2 Ⓑ 189 units^2

Ⓒ 224 units^2 Ⓓ 258 units^2

31. **ALGEBRA** For all integers n, $\boxed{n} = n^2$ if n is odd and $\boxed{n} = \sqrt{n}$ if n is even. What is the value of $\boxed{16} + \boxed{9}$?

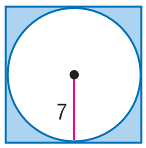
Ⓐ 7 Ⓑ 25 Ⓒ 85 Ⓓ 97

Maintain Your Skills

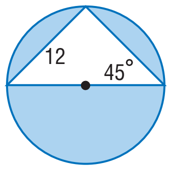
Mixed Review

Find the area of each shaded region. Assume that all polygons are regular unless otherwise stated. Round to the nearest tenth. *(Lesson 11-3)*

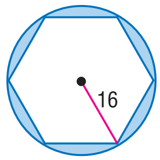
32.



33.



34.



Find the area of each figure. Round to the nearest tenth if necessary. *(Lesson 11-2)*

35. equilateral triangle with perimeter of 57 feet

36. rhombus with a perimeter of 40 yards and a diagonal of 12 yards

37. isosceles trapezoid with a perimeter of 90 meters if the longer base is 5 meters less than twice as long as the other base, each leg is 3 meters less than the shorter base, and the height is 15 meters

38. **COORDINATE GEOMETRY** The point (6, 0) is rotated 45° clockwise about the origin. Find the exact coordinates of its image. *(Lesson 9-3)*

Getting Ready for the Next Lesson

BASIC SKILL **Express each fraction as a decimal to the nearest hundredth.**

39. $\frac{5}{8}$ **40.** $\frac{13}{16}$ **41.** $\frac{9}{47}$ **42.** $\frac{10}{21}$

Practice Quiz 2

Lessons 11-3 and 11-4

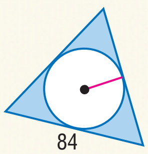
Find the area of each polygon. Round to the nearest tenth. *(Lesson 11-3)*

1. regular hexagon with apothem length of 14 millimeters

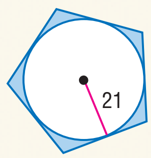
2. regular octagon with a perimeter of 72 inches

Find the area of each shaded region. Assume that all polygons are regular. Round to the nearest tenth. *(Lesson 11-3)*

3.



4.



5. **COORDINATE GEOMETRY** Find the area of *CDGHJ* with vertices $C(-3, -2)$, $D(1, 3)$, $G(5, 5)$, $H(8, 3)$, and $J(5, -2)$. *(Lesson 11-4)*

11-5 Geometric Probability

What You'll Learn

- Solve problems involving geometric probability.
- Solve problems involving sectors and segments of circles.

How can geometric probability help you win a game of darts?

To win at darts, you have to throw a dart at either the center or the part of the dartboard that earns the most points. In games, probability can sometimes be used to determine chances of winning. Probability that involves a geometric measure such as length or area is called **geometric probability**.

Vocabulary

- geometric probability
- sector
- segment

Study Tip

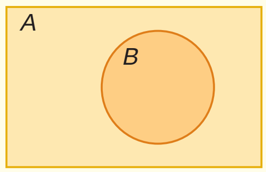
Look Back
To review **probability with line segments,** see page 20.

GEOMETRIC PROBABILITY In Chapter 1, you learned that the probability that a point lies on a part of a segment can be found by comparing the length of the part to the length of the whole segment. Similarly, you can find the probability that a point lies in a part of a figure by comparing the area of the part to the area of the whole figure.

Key Concept — *Probability and Area*

If a point in region A is chosen at random, then the probability $P(B)$ that the point is in region B, which is in the interior of region A, is

$$P(B) = \frac{\text{area of region } B}{\text{area of region } A}.$$



When determining geometric probability with targets, we assume

- that the object lands within the target area, and
- it is equally likely that the object will land anywhere in the region.

Standardized Test Practice

The Princeton Review
Test-Taking Tip
Analyze the Question
If allowed, circle or underline key words as you read the test question.

Example 1 *Probability with Area*

Grid In Test Item

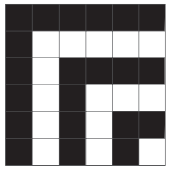
A square game board has black and white stripes of equal width as shown. What is the chance that a dart thrown at the board will land on a white stripe?

Read the Test Item

You want to find the probability of landing on a white stripe, not a black stripe.

Solve the Test Item

We need to divide the area of the white stripes by the total area of the game board. Extend the sides of each stripe. This separates the square into 36 small unit squares.



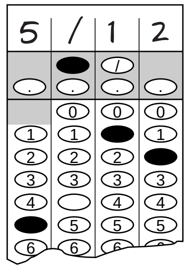
The white stripes have an area of 15 square units. The total area is 36 square units.

The probability of tossing a chip onto the white stripes is $\frac{15}{36}$ or $\frac{5}{12}$.

Fill in the Grid

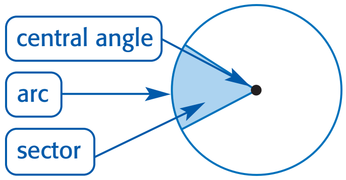
Write $\frac{5}{12}$ as 5/12 in the top row of the grid-in.

Then shade in the appropriate bubble under each entry.



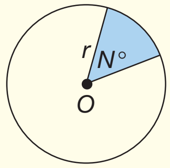
SECTORS AND SEGMENTS OF CIRCLES

Sometimes you need to know the area of a sector of a circle in order to find a geometric probability. A **sector** of a circle is a region of a circle bounded by a central angle and its intercepted arc.



Key Concept *Area of a Sector*

If a sector of a circle has an area of A square units, a central angle measuring $N°$, and a radius of r units, then $A = \frac{N}{360}\pi r^2$.



Study Tip

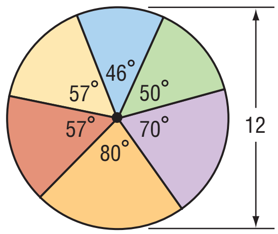
Common Misconceptions

The probability of an event can be expressed as a decimal or a fraction. These numbers are also sometimes represented by a percent.

Example 2 Probability with Sectors

a. Find the area of the blue sector.

Use the formula to find the area of the sector.



$A = \frac{N}{360}\pi r^2$ Area of a sector

$= \frac{46}{360}\pi(6^2)$ $N = 46$, $r = 6$

$= 4.6\pi$ Simplify.

b. Find the probability that a point chosen at random lies in the blue region.

To find the probability, divide the area of the sector by the area of the circle. The area of the circle is πr^2 with a radius of 6.

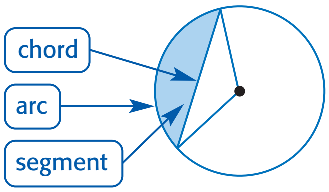
$P(\text{blue}) = \frac{\text{area of sector}}{\text{area of circle}}$ Geometric probability formula

$= \frac{4.6\pi}{\pi \cdot 6^2}$ Area of sector $= 4.6\pi$, area of circle $= \pi \cdot 6^2$

≈ 0.13 Use a calculator.

The probability that a random point is in the blue sector is about 0.13 or 13%.

The region of a circle bounded by an arc and a chord is called a **segment** of a circle. To find the area of a segment, subtract the area of the triangle formed by the radii and the chord from the area of the sector containing the segment.

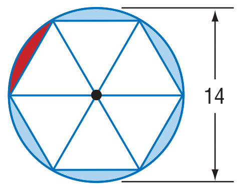


Example 3 Probability with Segments

A regular hexagon is inscribed in a circle with a diameter of 14.

a. Find the area of the red segment.

Area of the sector:



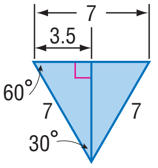
$A = \frac{N}{360}\pi r^2$ Area of a sector

$= \frac{60}{360}\pi(7^2)$ $N = 60, r = 7$

$= \frac{49}{6}\pi$ Simplify.

≈ 25.66 Use a calculator.

Area of the triangle:



Since the hexagon was inscribed in the circle, the triangle is equilateral, with each side 7 units long. Use properties of 30°-60°-90° triangles to find the apothem. The value of x is 3.5, the apothem is $x\sqrt{3}$ or $3.5\sqrt{3}$ which is approximately 6.06.

Next, use the formula for the area of a triangle.

$A = \frac{1}{2}bh$ Area of a triangle

$= \frac{1}{2}(7)(6.06)$ $b = 7, h = 6.06$

≈ 21.22 Simplify.

Area of the segment:

area of segment = area of sector − area of triangle

$\approx 25.66 - 21.22$ Substitution

≈ 4.44 Simplify.

b. Find the probability that a point chosen at random lies in the red region.

Divide the area of the sector by the area of the circle to find the probability. First, find the area of the circle. The radius is 7, so the area is $\pi(7^2)$ or about 153.94 square units.

$$P(\text{blue}) = \frac{\text{area of segment}}{\text{area of circle}}$$

$$\approx \frac{4.44}{153.94}$$

$$\approx 0.03$$

The probability that a random point is on the red segment is about 0.03 or 3%.

Check for Understanding

Concept Check

1. **Explain** how to find the area of a sector of a circle.
2. **OPEN ENDED** List three games that involve geometric probability.
3. **FIND THE ERROR** Rachel and Taimi are finding the probability that a point chosen at random lies in the green region.

Rachel

$A = \frac{N}{360}\pi r^2$

$= \frac{59 + 62}{360}\pi(5^2)$

≈ 26.4

$P(\text{green}) \approx \frac{26.4}{25\pi} \approx 0.34$

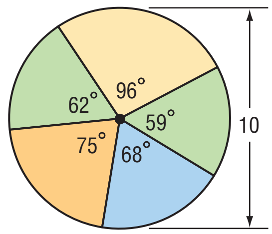
Taimi

$A = \frac{N}{360}\pi r^2$

$= \frac{59}{360}\pi(5^2) + \frac{62}{360}$

≈ 13.0

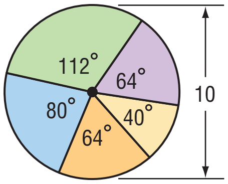
$P(\text{green}) \approx \frac{13.0}{25\pi} \approx 0.17$

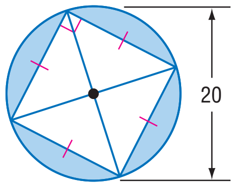


Who is correct? Explain your answer.

Guided Practice

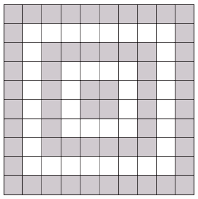
Find the area of the blue region. Then find the probability that a point chosen at random will be in the blue region.

4. 

5. 

Standardized Test Practice

6. What is the chance that a point chosen at random lies in the shaded region?



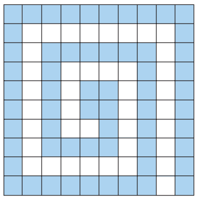
Practice and Apply

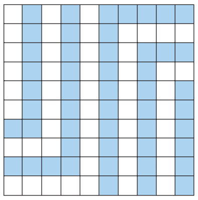
Homework Help

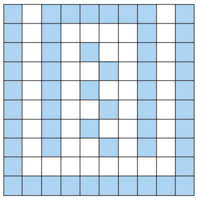
For Exercises	See Examples
7–9, 16, 24–30	1
10–15, 20–23	2
17–19	3

Extra Practice
See page 777.

Find the probability that a point chosen at random lies in the shaded region.

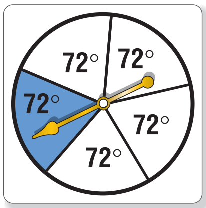
7. 

8. 

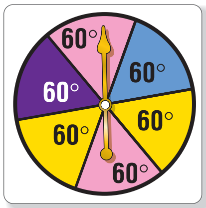
9. 

Find the area of the indicated sector. Then find the probability of spinning the color indicated if the diameter of each spinner is 15 centimeters.

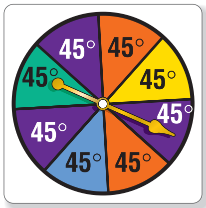
10. blue



11. pink

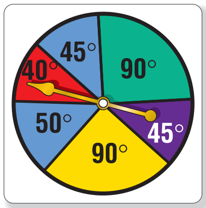


12. purple

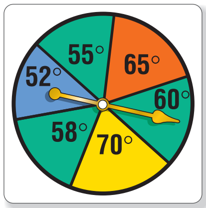


Find the area of the indicated sector. Then find the probability of choosing the color indicated if the diameter of each spinner is 15 centimeters.

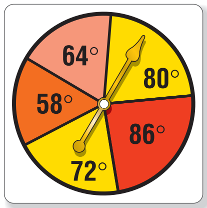
13. red



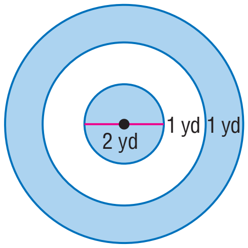
14. green



15. yellow

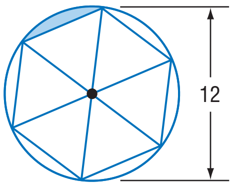


16. **PARACHUTES** A skydiver must land on a target of three concentric circles. The diameter of the center circle is 2 yards, and the circles are spaced 1 yard apart. Find the probability that she will land on the shaded area.

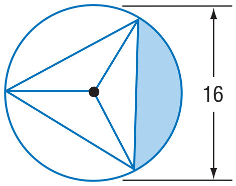


Find the area of the shaded region. Then find the probability that a point chosen at random is in the shaded region. Assume all inscribed polygons are regular.

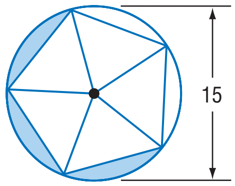
17.



18.

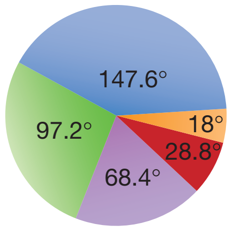


19.



SURVEYS **For Exercises 20–23, use the following information.**
A survey was taken at a high school, and the results were put in a circle graph. The students were asked to list their favorite colors. The measurement of each central angle is shown. If a person is chosen at random from the school, find the probability of each response.

What's Your Favorite Color?



20. Favorite color is red.
21. Favorite color is blue or green.
22. Favorite color is *not* red or blue.
23. Favorite color is *not* orange or green.

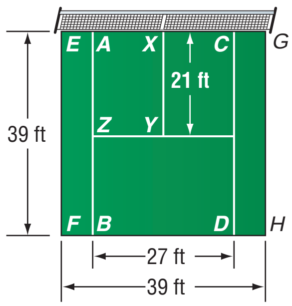
More About. . .

Tennis

In tennis, the linesman determines whether the hit ball is in or out. The umpire may only overrule the linesman if he or she immediately thinks the call was wrong without a doubt and never as a result of a player's protest.

Source: www.usta.com

TENNIS **For Exercises 24 and 25, use the following information.**
A tennis court has stripes dividing it into rectangular regions. For singles play, the inbound region is defined by segments $\overline{AB}$ and $\overline{CD}$. The doubles court is bound by the segments $\overline{EF}$ and $\overline{GH}$.



24. Find the probability that a ball in a singles game will land inside the court, but out of bounds.
25. When serving, the ball must land within $AXYZ$, the service box. Find the probability that a ball will land in the service box, relative to the court.

Stu Forster/Getty Images

DARTS **For Exercises 26–30, use the following information.**
Each sector of the dartboard has congruent central angles. Find the probability that the dart will land on the indicated color. The diameter of the center circle is 2 units.

26. black **27.** white **28.** red

29. Point values are assigned to each color. Should any of the colors have the same point value? Explain.

30. Which color should have the lowest point value? Explain.

31. **CRITICAL THINKING** Study each spinner in Exercises 13–15.
a. Are the chances of landing on each color equal? Explain.
b. Would this be considered a fair spinner to use in a game? Explain.

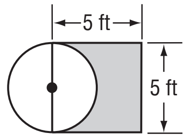
32. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can geometric probability help you win a game of darts?

Include the following in your answer:
- an explanation of how to find the geometric probability of landing on a red sector, and
- an explanation of how to find the geometric probability of landing in the center circle.

Standardized Test Practice

33. One side of a square is a diameter of a circle. The length of one side of the square is 5 feet. What is the probability that a point chosen at random is in the shaded region?



(A) 0.08 (B) 0.22 (C) 0.44 (D) 0.77

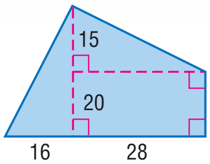
34. **ALGEBRA** If $4y = 16$, then $12 \div y =$
(A) 1. (B) 2. (C) 3. (D) 4.

Maintain Your Skills

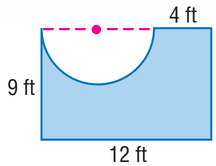
Mixed Review

Find the area of each figure. Round to the nearest tenth if necessary. *(Lesson 11-4)*

35.



36.



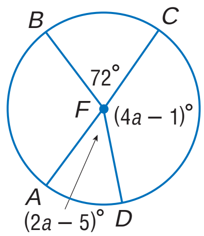
Find the area of each polygon. *(Lesson 11-3)*

37. a regular triangle with a perimeter of 48 feet

38. a square with a side length of 21 centimeters

39. a regular hexagon with an apothem length of 8 inches

ALGEBRA **Find the measure of each angle on ⊙F with diameter $\overline{AC}$.** *(Lesson 10-2)*



40. $\angle AFB$ **41.** $\angle CFD$ **42.** $\angle AFD$ **43.** $\angle DFB$

Find the length of the third side of a triangle given the measures of two sides and the included angle of the triangle. Round to the nearest tenth. *(Lesson 7-7)*

44. $m = 6.8, n = 11.1, m\angle P = 57$

45. $f = 32, h = 29, m\angle G = 41$

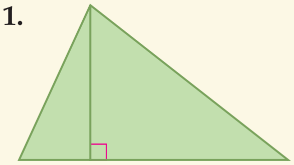
Chapter 11 Study Guide and Review

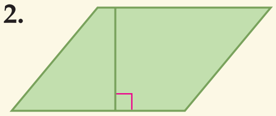
Vocabulary and Concept Check

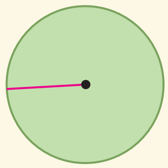
apothem (p. 610)
geometric probability (p. 622)
irregular figure (p. 617)
irregular polygon (p. 618)
sector (p. 623)
segment (p. 624)

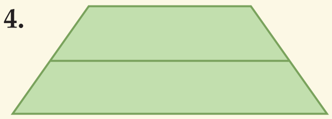
A complete list of postulates and theorems can be found on pages R1–R8.

Exercises **Choose the formula to find the area of each shaded figure.**

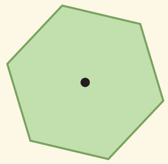
1. 

2. 

3. 

4. 

5. 

6. 

a. $A = \pi r^2$

b. $A = \frac{N}{360}\pi r^2$

c. $A = \frac{1}{2}bh$

d. $A = \frac{1}{2}Pa$

e. $A = bh$

f. $A = \frac{1}{2}h(b_1 + b_2)$

Lesson-by-Lesson Review

11-1 Area of Parallelograms

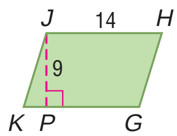
See pages 595–600.

Concept Summary

- The area of a parallelogram is the product of the base and the height.

Example **Find the area of ▱*GHJK*.**

The area of a parallelogram is given by the formula $A = bh$.

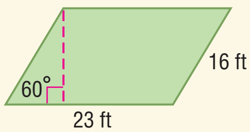


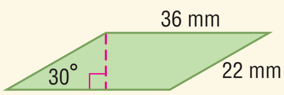
$A = bh$ Area of a parallelogram

$= 14(9)$ or 126 $b = 14, h = 9$

The area of the parallelogram is 126 square units.

Exercises **Find the perimeter and area of each parallelogram.** *See Example 1 on page 596.*

7. 

8. 

COORDINATE GEOMETRY **Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral.** *See Example 3 on page 597.*

9. $A(-6, 1), B(1, 1), C(1, -6), D(-6, -6)$

10. $E(7, -2), F(1, -2), G(2, 2), H(8, 2)$

11. $J(-1, -4), K(-5, 0), L(-5, 5), M(-1, 1)$

12. $P(-7, -1), Q(-3, 3), R(-1, 1), S(-5, -3)$

www.geometryonline.com/vocabulary_review

CONTENTS

11-2 Areas of Triangles, Rhombi, and Trapezoids

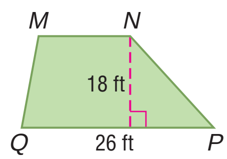
See pages 601–609.

Concept Summary

- The formula for the area of a triangle can be used to find the areas of many different figures.
- Congruent figures have equal areas.

Example **Trapezoid $MNPQ$ has an area of 360 square feet. Find the length of $\overline{MN}$.**

$A = \frac{1}{2}h(b_1 + b_2)$	Area of a trapezoid
$360 = \frac{1}{2}(18)(b_1 + 26)$	$A = 360$, $h = 18$, $b_2 = 26$
$360 = 9b_1 + 234$	Multiply.
$14 = b_1$	Solve for b_1.

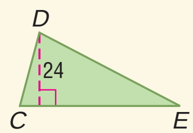


The length of $\overline{MN}$ is 14 feet.

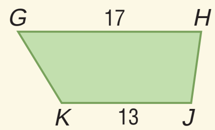
Exercises **Find the missing measure for each quadrilateral.** *See Example 4 on page 604.*

13. Triangle CDE has an area of 336 square inches. Find CE.

14. Trapezoid $GHJK$ has an area of 75 square meters. Find the height.



Exercise 13



Exercise 14

11-3 Areas of Regular Polygons and Circles

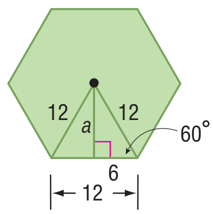
See pages 610–616.

Concept Summary

- A regular n-gon is made up of n congruent isosceles triangles.
- The area of a circle of radius r units is πr^2 square units.

Example **Find the area of a regular hexagon with a perimeter of 72 feet.**

Since the perimeter is 72 feet, the measure of each side is 12 feet. The central angle of a hexagon is 60°. Use the properties of 30°-60°-90° triangles to find that the apothem is $6\sqrt{3}$ feet.



$A = \frac{1}{2}Pa$	Area of a regular polygon
$= \frac{1}{2}(72)(6\sqrt{3})$	$P = 72$, $a = 6\sqrt{3}$
$= 216\sqrt{3}$	Simplify.
≈ 374.1	

The area of the regular hexagon is 374.1 square feet to the nearest tenth.

Exercises **Find the area of each polygon. Round to the nearest tenth.**
See Example 1 on page 611.

15. a regular pentagon with perimeter of 100 inches

16. a regular decagon with side length of 12 millimeters

Chapter 11 For More ...

- Extra Practice, see pages 776–777.
- Mixed Problem Solving, see page 792.

11-4 Areas of Irregular Figures

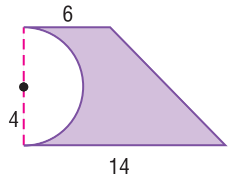
See pages 617–621.

Concept Summary

- The area of an irregular figure is the sum of the areas of its nonoverlapping parts.

Example

Find the area of the figure.



Separate the figure into a rectangle and a triangle.

area of irregular figure = area of rectangle − area of semicircle + area of triangle

$= \ell w - \frac{1}{2}\pi r^2 + \frac{1}{2}bh$ Area formulas

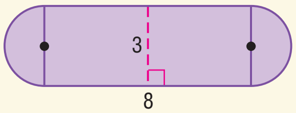
$= (6)(8) - \frac{1}{2}\pi(4^2) + \frac{1}{2}(8)(8)$ Substitution

$= 48 - 8\pi + 32$ or about 54.9 Simplify.

The area of the irregular figure is 54.9 square units to the nearest tenth.

Exercises **Find the area of each figure to the nearest tenth.** *See Example 1 on page 617.*

17.



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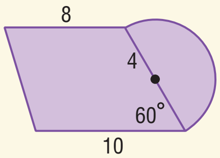


11-5 Geometric Probability

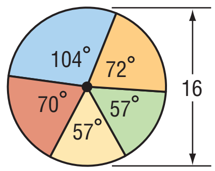
See pages 622–627.

Concept Summary

- To find a geometric probability, divide the area of a part of a figure by the total area.

Example

Find the probability that a point chosen at random will be in the blue sector.



First find the area of the blue sector.

$A = \frac{N}{360}\pi r^2$ Area of a sector

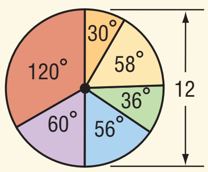
$= \frac{104}{360}\pi(8^2)$ or about 58.08 Substitute and simplify.

To find the probability, divide the area of the sector by the area of the circle.

$P(\text{blue}) = \frac{\text{area of sector}}{\text{area of circle}}$ Geometric probability formula

$= \frac{58.08}{\pi 8^2}$ or about 0.29 The probability is about 0.29 or 29%.

Exercises **Find the probability that a point chosen at random in the figure is the given color.** *See Example 2 on page 623.*



19. red

20. purple or green

Chapter 11 Practice Test

Vocabulary and Concepts

Choose the letter of the correct area formula for each figure.

1. regular polygon
2. trapezoid
3. triangle

a. $A = \frac{1}{2}Pa$

b. $A = \frac{1}{2}bh$

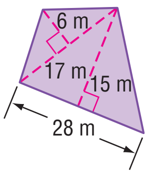
c. $A = \frac{1}{2}h(b_1 + b_2)$

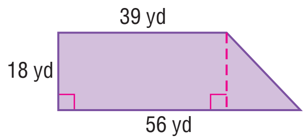
Skills and Applications

COORDINATE GEOMETRY **Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral.**

4. $R(-6, 8)$, $S(-1, 5)$, $T(-1, 1)$, $U(-6, 4)$
5. $R(7, -1)$, $S(9, 3)$, $T(5, 5)$, $U(3, 1)$
6. $R(2, 0)$, $S(4, 5)$, $T(7, 5)$, $U(5, 0)$
7. $R(3, -6)$, $S(9, 3)$, $T(12, 1)$, $U(6, -8)$

Find the area of each figure. Round to the nearest tenth if necessary.

8.


9.


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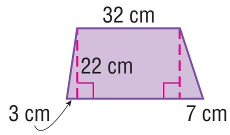
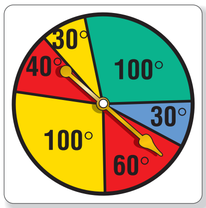
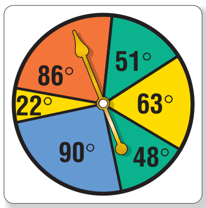
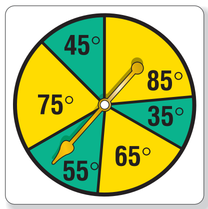


11. a regular octagon with apothem length of 3 ft
12. a regular pentagon with a perimeter of 115 cm

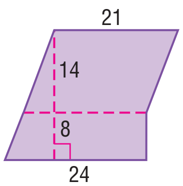
Each spinner has a diameter of 12 inches. Find the probability of spinning the indicated color.

13. red


14. orange


15. green


Find the area of each figure. Round to the nearest tenth.

16.


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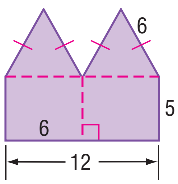

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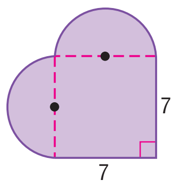


19. **SOCCER BALLS** The surface of a soccer ball is made of a pattern of regular pentagons and hexagons. If each hexagon on a soccer ball has a perimeter of 9 inches, what is the area of a hexagon?

20. **STANDARDIZED TEST PRACTICE** What is the area of a quadrilateral with vertices at $(-3, -1)$, $(-1, 4)$, $(7, 4)$, and $(5, -1)$?

Ⓐ 50 units^2 Ⓑ 45 units^2 Ⓒ $8\sqrt{29}$ units^2 Ⓓ 40 units^2

Chapter 11 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Solve $3\left(\frac{2x - 4}{-6}\right) = 18.$ (Prerequisite Skill)

Ⓐ -19 Ⓑ -16 Ⓒ 4 Ⓓ 12

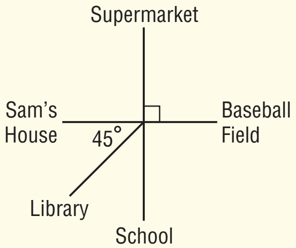
2. Sam rode his bike along the path from the library to baseball practice. What type of angle did he form during the ride? (Lesson 1-5)

Ⓐ straight

Ⓑ obtuse

Ⓒ acute

Ⓓ right



3. What is the logical conclusion of these statements?

If you exercise, you will maintain better health.
If you maintain better health, you will live longer.
(Lesson 2-4)

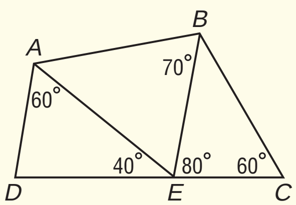
Ⓐ If you exercise, you will live longer.

Ⓑ If you do not exercise, you will not live longer.

Ⓒ If you do not exercise, you will not maintain better health.

Ⓓ If you maintain better health, you will not live longer.

4. Which segments are parallel? (Lesson 3-5)



Ⓐ $\overline{AB}$ and $\overline{CD}$ Ⓑ $\overline{AD}$ and $\overline{BC}$

Ⓒ $\overline{AD}$ and $\overline{BE}$ Ⓓ $\overline{AE}$ and $\overline{BC}$

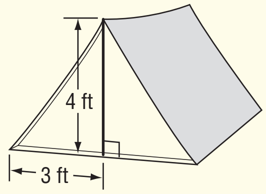
5. The front view of a pup tent resembles an isosceles triangle. The entrance to the tent is an angle bisector. The tent is secured by stakes. What is the distance between the two stakes? (Lesson 5-1)

Ⓐ 3 ft

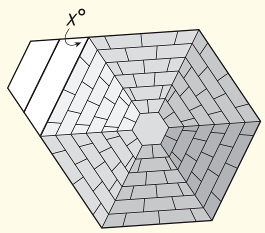
Ⓑ 4 ft

Ⓒ 5 ft

Ⓓ 6 ft



6. A carpenter is building steps leading to a hexagonal gazebo. The outside edges of the steps need to be cut at an angle. Find *x*. (Lesson 8-1)



Ⓐ 180 Ⓑ 120 Ⓒ 72 Ⓓ 60

7. Which statement is *always* true? (Lesson 10-4)

Ⓐ When an angle is inscribed in a circle, the angle's measure equals one-half of the measure of the intercepted arc.

Ⓑ In a circle, an inscribed quadrilateral will have consecutive angles that are supplementary.

Ⓒ In a circle, an inscribed angle that intercepts a semicircle is obtuse.

Ⓓ If two inscribed angles of a circle intercept congruent arcs, then the angles are complementary.

8. The apothem of a regular hexagon is 7.8 centimeters. If the length of each side is 9 centimeters, what is the area of the hexagon? (Lesson 11-3)

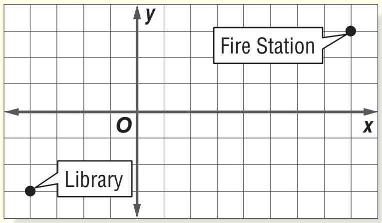
Ⓐ 35.1 cm^2 Ⓑ 70.2 cm^2

Ⓒ 210.6 cm^2 Ⓓ 421.2 cm^2

Part 2 Short Response/Grid In

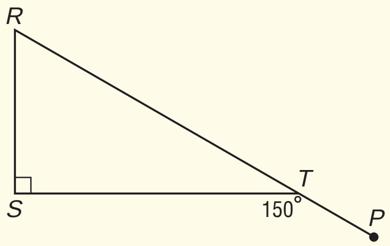
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. The post office is located halfway between the fire station and the library. What are the coordinates of the post office? (Lesson 1-3)

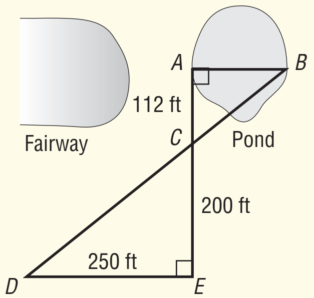


10. What is the slope of a line perpendicular to the line represented by the equation $3x - 6y = 12$? (Lesson 3-3)

11. $\triangle RST$ is a right triangle. Find $m\angle R$. (Lesson 4-2)



12. If $\angle A$ and $\angle E$ are congruent, find AB, the distance in feet across the pond. (Lesson 6-3)



13. If point $J(6, -3)$ is translated 5 units up and then reflected over the y-axis, what will the new coordinates of J' be? (Lesson 9-2)

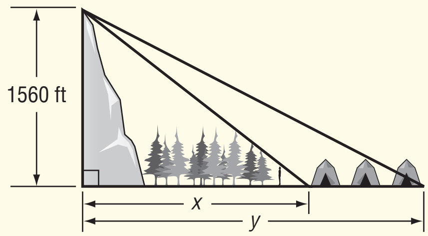
The Princeton Review Test-Taking Tip

Question 4 Multi-Step Problems To find the pair of parallel lines, first you need to find the missing angle measures. Use the Angle Sum Theorem to find the measures of the angles in each triangle.

Part 3 Open Ended

Record your answers on a sheet of paper. Show your work.

14. Lori and her family are camping near a mountain. Their campground is in a clearing next to a stretch of forest.



a. The angle of elevation from Lori's line of sight at the edge of the forest to the top of the mountain, is 38°. Find the distance x from the base of the mountain to the edge of the forest. Round to the nearest foot. (Lesson 7-5)

b. The angle of elevation from the far edge of the campground to the top of the mountain is 35°. Find the distance y from the base of the mountain to the far edge of the campground. Round to the nearest foot. (Lesson 7-5)

c. What is the width of the campground? Round to the nearest foot. (Lesson 7-5)

15. Parallelogram $ABCD$ has vertices $A(0, 0)$, $B(3, 4)$, and $C(8, 4)$.

a. Find the possible coordinates for D. (Lesson 8-2)

b. Find the area of $ABCD$. (Lesson 11-1)