Collecting and analyzing data allows you to make decisions and predictions about the future. In this unit, you will learn about statistics and probability.
The U.S. government has been counting each person in the country since its first Census following independence was taken in 1790. Befitting the first Census of the 21st century, the Census Bureau allowed Census 2000 questionnaires to be completed electronically for the first time. In this project, you will see how data analysis can be used to compare statistics about a state of your choice to other states in the United States.

Log on to www.algebra1.com/webquest. Begin your WebQuest by reading the Task.

Then continue working on your WebQuest as you study Unit 5.
Each day statistics are reported in the newspapers, in magazines, on television, and on the radio. These data involve business, government, ecology, sports, and many other topics. A basic knowledge of statistics allows you to interpret what you hear and read in the media. One important tool to help you understand the significance of a set of data is the box-and-whisker plot. You will draw and use a box-and-whisker plot for data involving NASCAR racing in Lesson 13-5.
Prerequisite Skills  To be successful in this chapter, you’ll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 13.

For Lesson 13-1  Use Logical Reasoning
Find a counterexample for each statement.  (For review, see Lesson 1-7.)

1. If $a + b = c$, then $a < c$.
2. If a flower is a rose, then it is red.
3. If Tara obeys the speed limit, then she will drive 45 miles per hour or less.
4. If a number is even, then it is divisible by 4.

For Lesson 13-4  Find the Median
Find the median for each set of data.  (For review, see pages 818 and 819.)

5. 1, 7, 9, 15, 25, 59, 63
6. 0, 10, 2, 2, 9, 5, 4, 2, 8, 3, 8, 7, 3

For Lesson 13-5  Graph Numbers on a Number Line
Graph each set of numbers on a number line.  (For review, see Lesson 2-1.)

8. {7, 9, 10, 13, 14}
9. {15, 17.5, 19, 20.5, 23}
10. {3.2, 4.8, 5.0, 5.7, 6.1}
11. {2.3, 2.8, 3.1, 3.7, 4.5}

Foldables™

Make this Foldable to help you organize information about statistics. Begin with three sheets of plain 8 1/2" by 11" paper.

Step 1  Stack Pages
Stack sheets of paper with edges 3/4 inch apart.

Step 2  Fold Up Bottom Edges
All tabs should be the same size.

Step 3  Crease and Staple
Staple along fold.

Step 4  Turn and Label
Label the tabs with topics from the chapter.

Reading and Writing  As you read and study the chapter, use each page to write notes and examples.
What You’ll Learn

- Identify various sampling techniques.
- Recognize a biased sample.

Why is sampling important in manufacturing?

Manufacturing music CDs involves burning, or recording, copies from a master. However, not every burn is successful. It is costly and time-consuming to check every CD that is burned. Therefore, in order to monitor production, some CDs are picked at random and checked for defects.

Sampling Techniques When you wish to make an investigation, there are four ways that you can collect data.

- Published data Use data that are already in a source like a newspaper or book.
- Observational study Watch naturally occurring events and record the results.
- Experiment Conduct an experiment and record the results.
- Survey Ask questions of a group of people and record the results.

When performing an experiment or taking a survey, researchers often choose a sample. A sample is some portion of a larger group, called the population, selected to represent that group. If all of the units within a population are included, it is called a census. Sample data are often used to estimate a characteristic within an entire population, such as voting preferences prior to elections.

### Random Samples

<table>
<thead>
<tr>
<th>Type</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple Random Sample</td>
<td>A simple random sample is a sample that is as likely to be chosen as any other from the population.</td>
<td>The 26 students in a class are each assigned a different number from 1 to 26. Then three of the 26 numbers are picked at random.</td>
</tr>
<tr>
<td>Stratified Random Sample</td>
<td>In a stratified random sample, the population is first divided into similar, nonoverlapping groups. A simple random sample is then selected from each group.</td>
<td>The students in a school are divided into freshman, sophomores, juniors, and seniors. Then two students are randomly selected from each group of students.</td>
</tr>
<tr>
<td>Systematic Random Sample</td>
<td>In a systematic random sample, the items are selected according to a specified time or item interval.</td>
<td>Every 2 minutes, an item is pulled off the assembly line. or Every twentieth item is pulled off the assembly line.</td>
</tr>
</tbody>
</table>

A random sample of a population is selected so that it is representative of the entire population. The sample is chosen without any preference. There are several ways to pick a random sample.
Example 1 Classify a Random Sample

ECOLOGY Ten lakes are selected randomly from a list of all public-access lakes in Minnesota. Then 2 liters of water are drawn from 20 feet deep in each of the ten lakes.

a. Identify the sample and suggest a population from which it was selected.
The sample is ten 2-liter containers of lake water, one from each of 10 lakes. The population is lake water from all of the public-access lakes in Minnesota.

b. Classify the sample as simple, stratified, or systematic.
This is a simple random sample. Each of the ten lakes was equally likely to have been chosen from the list.

Example 2 Identify Sample as Biased or Unbiased

Identify each sample as biased or unbiased. Explain your reasoning.

a. MANUFACTURING Every 1000th bolt is pulled from the production line and measured for length.
The sample is chosen using a specified time interval. This is an unbiased sample because it is a systematic random sample.

b. MUSIC Every tenth customer in line for a certain rock band’s concert tickets is asked about his or her favorite rock band.
The sample is a biased sample because customers in line for concert tickets are more likely to name the band giving the concert as a favorite band.

Example 3 Identify and Classify a Biased Sample

BUSINESS The travel account records from 4 of the 20 departments in a corporation are to be reviewed. The accountant states that the first 4 departments to voluntarily submit their records will be reviewed.

a. Identify the sample and suggest a population from which it was selected.
The sample is the travel account records from 4 departments in the corporation. The population is the travel account records from all 20 departments in the corporation.
b. Classify the sample as *convenience* or *voluntary response*.

Since the departments voluntarily submit their records, this is a voluntary response sample.

---

**Example 4** Identify the Sample

**NEWS REPORTING** For an article in the school paper, Rafael needs to determine whether students in his school believe that an arts center should be added to the school. He polls 15 of his friends who sing in the choir. Twelve of them think the school needs an arts center, so Rafael reports that 80% of the students surveyed support the project.

a. Identify the sample.

The sample is a group of students from the choir.

b. Suggest a population from which the sample was selected.

The population for the survey is all of the students in the school.

c. State whether the sample is *unbiased* (random) or *biased*. If unbiased, classify it as *simple*, *stratified*, or *systematic*. If biased, classify it as *convenience* or *voluntary response*.

The sample was not randomly selected from the entire student body. So the reported support is not likely to be representative of the student body. The sample is biased. Since Rafael polled only his friends, it is a convenience sample.

---

**Check for Understanding**

**Concept Check**

1. Describe how the following three types of sampling techniques are similar and how they are different.
   - simple random sample
   - stratified random sample
   - systematic random sample

2. Explain the difference between a convenience sample and a voluntary response sample.

3. **OPEN ENDED** Give an example of a biased sample.

**Guided Practice**

Identify each sample, suggest a population from which it was selected, and state whether it is *unbiased* (random) or *biased*. If unbiased, classify the sample as *simple*, *stratified*, or *systematic*. If biased, classify as *convenience* or *voluntary response*.

4. **NEWSPAPERS** The local newspaper asks readers to write letters stating their preferred candidate for mayor.

5. **SCHOOL** A teacher needs a sample of work from 4 students in her first-period math class to display at the school open house. She selects the work of the first 4 students who raise their hands.

6. **BUSINESS** A hardware store wants to assess the strength of nails it sells. Store personnel select 25 boxes at random from among all of the boxes on the shelves. From each of the 25 boxes, they select one nail at random and subject it to a strength test.

7. **SCHOOL** A class advisor hears complaints about an incorrect spelling of the school name on pencils sold at the school store. The advisor goes to the store and asks Namid to gather a sample of pencils and look for spelling errors. Namid grabs the closest box of pencils and counts out 12 pencils from the top of the box. She checks the pencils, returns them to the box, and reports the results to the advisor.
Identify each sample, suggest a population from which it was selected, and state whether it is unbiased (random) or biased. If unbiased, classify the sample as simple, stratified, or systematic. If biased, classify as convenience or voluntary response.

8. **SCHOOL**  Pieces of paper with the names of 3 sophomores are drawn from a hat containing identical pieces of paper with all sophomores’ names.

9. **FOOD**  Twenty shoppers outside a fast-food restaurant are asked to name their preferred cola among two choices.

10. **RECYCLING**  An interviewer goes from house to house on weekdays between 9 A.M. and 4 P.M. to determine how many people recycle.

11. **POPULATION**  A state is first divided into its 86 counties and then 10 people from each county are chosen at random.

12. **SCOOTERS**  A scooter manufacturer is concerned about quality control. The manufacturer checks the first 5 scooters off the line in the morning and the last 5 off the line in the afternoon for defects.

13. **SCHOOL**  To determine who will speak for her class at the school board meeting, Ms. Finchie used the numbers appearing next to her students’ names in her grade book. She writes each of the numbers on an identical piece of paper and shuffles the pieces of papers in a box. Without seeing the contents of the box, one student draws 3 pieces of paper from the box. The students whose numbers match the numbers chosen will speak for the class.

14. **FARMING**  An 8-ounce jar was filled with corn from a storage silo by dipping the jar into the pile of corn. The corn in the jar was then analyzed for moisture content.

15. **COURT**  The gender makeup of district court judges in the United States is to be estimated from a sample. All judges are grouped geographically by federal reserve districts. Within each of the 11 federal reserve districts, all judges’ names are assigned a distinct random number. In each district, the numbers are then listed in order. A number between 1 and 20 inclusive is selected at random, and the judge with that number is selected. Then every 20th name after the first selected number is also included in the sample.

16. **TELEVISION**  A television station asks its viewers to share their opinions about a proposed golf course to be built just outside the city limits. Viewers can call one of two 900-numbers. One number represents a “yes” vote, and the other number represents a “no” vote.

17. **GOVERNMENT**  To discuss leadership issues shared by all United States Senators, the President asks 4 of his closest colleagues in the Senate to meet with him.

18. **FOOD**  To sample the quality of the Bing cherries throughout the produce department, the produce manager picks up a handful of cherries from the edge of one case and checks to see if these cherries are spoiled.

19. **MANUFACTURING**  During the manufacture of high-definition televisions, units are checked for defects. Within the first 10 minutes of a work shift, a television is randomly chosen from the line of completed sets. For the rest of the shift, every 15th television on the line is checked for defects.

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More About...

**Food**

Michigan leads the nation in cherry production by growing about 219 million pounds of cherries per year.

Source: World Book Encyclopedia
Identify each sample, suggest a population from which it was selected, and state whether it is unbiased (random) or biased. If unbiased, classify the sample as simple, stratified, or systematic. If biased, classify as convenience or voluntary response.

20. BUSINESS To get reaction about a benefits package, a company uses a computer program to randomly pick one person from each of its departments.

21. MOVIES A magazine is trying to determine the most popular actor of the year. It asks its readers to mail the name of their favorite actor to the magazine’s office.

COLLEGE For Exercises 22 and 23, use the following information.
The graph at the right reveals that 56% of survey respondents did not have a formal financial plan for a child’s college tuition.

22. Write a statement to describe what you do know about the sample.

23. What additional information would you like to have about the sample to determine whether the sample is biased?

24. SCHOOL Suppose you want to sample the opinion of the students in your school about a new dress code. Describe an unbiased way to conduct your survey.

25. ELECTIONS Suppose you are running for mayor of your city and want to know if you are likely to be elected. Describe an unbiased way to poll the voters.

26. FAMILY Study the graph at the right. Describe the information that is revealed in the graph. What information is there about the type or size of the sample?

27. FARMING Suppose you are a farmer and want to know if your tomato crop is ready to harvest. Describe an unbiased way to determine whether the crop is ready to harvest.

28. MANUFACTURING Suppose you want to know whether the infant car seats manufactured by your company meet the government standards for safety. Describe an unbiased way to determine whether the seats meet the standards.

29. CRITICAL THINKING The following is a proposal for surveying a stratified random sample of the student body.

Divide the student body according to those who are on the basketball team, those who are in the band, and those who are in the drama club. Then take a simple random sample from each of the three groups. Conduct the survey using this sample.

Study the proposal. Describe its strengths and weaknesses. Is the sample a stratified random sample? Explain.
30. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

Why is sampling important in manufacturing?
Include the following in your answer:
- an unbiased way to pick which CDs to check, and
- a biased way to pick which CDs to check.

31. To predict the candidate who will win the seat in city council, which method would give the newspaper the most accurate result?
   - A) Ask every 5th person that passes a reporter in the mall.
   - B) Use a list of registered voters and call every 20th person.
   - C) Publish a survey and ask readers to reply.
   - D) Ask reporters at the newspaper.

32. A cookie manufacturer plans to make a new type of cookie and wants to know if people will buy these cookies. For accurate results, which method should they use?
   - A) Ask visitors to their factory to evaluate the cookie.
   - B) Place a sample of the new cookie with their other cookies, and ask people to answer a questionnaire about the cookie.
   - C) Take samples to a school, and ask students to raise their hands if they like the cookie.
   - D) Divide the United States into 6 regions. Then pick 3 cities in each region at random, and conduct a taste test in each of the 18 cities.

33. Solve each equation. *(Lesson 12-9)*
   
   33. \( \frac{10}{3y} - \frac{5}{2y} = \frac{1}{4} \)
   
   34. \( \frac{3}{r + 4} - \frac{1}{r} = \frac{1}{r} \)
   
   35. \( \frac{1}{4m} + \frac{2m}{m - 3} = 2 \)

36. Simplify. *(Lesson 12-8)*
   
   36. \( \frac{2 + \frac{5}{x}}{\frac{x}{3} + \frac{5}{6}} \)
   
   37. \( \frac{a + \frac{35}{a + 12}}{a + 7} \)
   
   38. \( \frac{t^2 - 4}{t^2 + 5t + 6} \)

39. **GEOMETRY** What is the perimeter of \( \triangle ABC \)? *(Lesson 11-2)*
   
   The perimeter is \( 11\sqrt{24} \) cm.

40. Solve each equation by using the Quadratic Formula. Approximate any irrational roots to the nearest tenth. *(Lesson 10-4)*
   
   40. \( x^2 - 6x - 40 = 0 \)
   
   41. \( 6b^2 + 15 = -19b \)
   
   42. \( 2d^2 = 9d + 3 \)

43. Find each product. *(Lesson 8-7)*
   
   43. \( (y + 5)(y + 7) \)
   
   44. \( (c - 3)(c - 7) \)
   
   45. \( (x + 4)(x - 8) \)

46. **BASIC SKILL** Find each sum or difference.
   
   46. \( 4.5 + 3.8 \)
   
   47. \( 16.9 + 7.21 \)
   
   48. \( 3.6 + 18.5 \)
   
   49. \( 7.6 - 3.8 \)
   
   50. \( 18 - 4.7 \)
   
   51. \( 13.2 - 0.75 \)
Investigating Slope-Intercept Form

Even though taking a random sample eliminates bias or favoritism in the choice of a sample, questions may be worded to influence people’s thoughts in a desired direction. Two different surveys on Internet sales tax had different results.

Question 1
Should there be sales tax on purchases made on the Internet?

Question 2
Do you think people should or should not be required to pay the same sales tax for purchases made over the Internet that they would if they had bought the item in person at a local store?

Notice the difference in Questions 1 and 2. Question 2 includes more information. Pointing out that customers pay sales tax for items bought at a local store may give the people answering the survey a reason to answer “yes.” Asking the question in that way probably led people to answer the way they did.

Because they are random samples, the results of both of these surveys are accurate. However, the results could be used in a misleading way by someone with an interest in the issue. For example, an Internet retailer would prefer to state the results of Question 1. Be sure to think about survey questions carefully to make sure that you interpret the results correctly.

Reading to Learn

For Exercises 1–2, tell whether each question is likely to bias the results. Explain your reasoning.

1. On a survey on environmental issues:
   a. “Due to diminishing resources, should a law be made to require recycling?”
   b. “Should the government require citizens to participate in recycling efforts?”

2. On a survey on education:
   a. “Should schools fund extracurricular sports programs?”
   b. “The budget of the River Valley School District is short of funds. Should taxes be raised in order for the district to fund extracurricular sports programs?”

3. Suppose you want to determine whether to serve hamburgers or pizza at the class party.
   a. Write a survey question that would likely produce biased results.
   b. Write a survey question that would likely produce unbiased results.
13-2 Introduction to Matrices

What You’ll Learn

• Organize data in matrices.

• Solve problems by adding or subtracting matrices or by multiplying by a scalar.

Vocabulary

• matrix
• dimensions
• row
• column
• element
• scalar multiplication

How are matrices used to organize data?

To determine the best type of aircraft to use for certain flights, the management of an airline company considers the following aircraft operating statistics.

The table has rows and columns of information. When we concentrate only on the numerical information, we see an array with 4 rows and 5 columns.

<table>
<thead>
<tr>
<th>Aircraft</th>
<th>Number of Seats</th>
<th>Airborne Speed (mph)</th>
<th>Possible Flight Distance (miles)</th>
<th>Fuel per Hour (gallons)</th>
<th>Operating Cost per Hour (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B747-100</td>
<td>462</td>
<td>512</td>
<td>2297</td>
<td>3517</td>
<td>7224</td>
</tr>
<tr>
<td>DC-10-10</td>
<td>297</td>
<td>496</td>
<td>1402</td>
<td>2311</td>
<td>5703</td>
</tr>
<tr>
<td>MD-11</td>
<td>259</td>
<td>527</td>
<td>3073</td>
<td>2464</td>
<td>6539</td>
</tr>
<tr>
<td>A300-600</td>
<td>228</td>
<td>475</td>
<td>1372</td>
<td>1505</td>
<td>4783</td>
</tr>
</tbody>
</table>

Source: Air Transport Association of America

This array of numbers is called a matrix.

Organize data in matrices

If you have ever used a spreadsheet program on the computer, you have worked with matrices. A matrix is a rectangular arrangement of numbers in rows and columns. A matrix is usually described by its dimensions, or the number of rows and columns, with the number of rows stated first. Each entry in a matrix is called an element.

Example 1 Name Dimensions of Matrices

State the dimensions of each matrix. Then identify the position of the circled element in each matrix.

a. \[
\begin{bmatrix}
11 & 15 & 24
\end{bmatrix}
\]

This matrix has 1 row and 3 columns. Therefore, it is a 1-by-3 matrix. The circled element is in the first row and the second column.

b. \[
\begin{bmatrix}
-4 & 2 \\
0 & 1 \\
3 & -6
\end{bmatrix}
\]

This matrix has 3 rows and 2 columns. Therefore, it is a 3-by-2 matrix. The circled element is in the third row and the first column.
Two matrices are equal only if they have the same dimensions and each element of one matrix is equal to the corresponding element in the other matrix.

\[
\begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix} \quad \begin{bmatrix} 2 & 4 \\ 1 & 7 \end{bmatrix} \neq \begin{bmatrix} 2 & 3 \\ 1 & 7 \end{bmatrix} \quad \begin{bmatrix} 4 & 8 \\ 1 & -3 \end{bmatrix} \neq \begin{bmatrix} 4 & 8 \\ 1 & -3 \end{bmatrix}
\]

**Matrix Operations** If two matrices have the same dimensions, you can add or subtract them. To do this, add or subtract corresponding elements of the two matrices.

**Example 2** Add Matrices

If \( A = \begin{bmatrix} 3 & -4 \\ -1 & 6 \end{bmatrix}, B = \begin{bmatrix} 7 & -4 \\ 1 & 6 \end{bmatrix} \) and \( C = \begin{bmatrix} 3 & 6 \\ -4 & 5 \end{bmatrix} \) find each sum.

If the sum does not exist, write impossible.

\( a. \ A + B \)

\[
A + B = \begin{bmatrix} 3 & -4 \\ -1 & 6 \end{bmatrix} + \begin{bmatrix} 7 & -4 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 3 + 7 & -4 + (-4) \\ -1 + 1 & 6 + 6 \end{bmatrix} + (-2) \]

\[
= \begin{bmatrix} 10 & -8 \\ 0 & 12 \end{bmatrix}
\]

\( b. \ B + C \)

\[
B + C = \begin{bmatrix} 7 & -4 \\ 1 & 6 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -4 & 5 \end{bmatrix} = \begin{bmatrix} 7 + 3 & -4 + 6 \\ 1 - 4 & 6 - 3 \end{bmatrix}
\]

\[
= \begin{bmatrix} 10 & 2 \\ -3 & 3 \end{bmatrix}
\]

Since \( B \) is a 2-by-3 matrix and \( C \) is a 2-by-2 matrix, the matrices do not have the same dimensions. Therefore, it is impossible to add these matrices.

Addition and subtraction of matrices can be used to solve real-world problems.

**Example 3** Subtract Matrices

**College Football**

Each year the National Football Foundation awards the MacArthur Bowl to the number one college football team. The bowl is made of about 400 ounces of silver and represents a stadium with rows of seats.

Source: ESPN Information Please® Sports Almanac

The Division I-A college football teams with the five best records during the 1990s are listed below.

<table>
<thead>
<tr>
<th>Overall Record</th>
<th>Bowl Record</th>
</tr>
</thead>
<tbody>
<tr>
<td>Florida State</td>
<td>Florida State</td>
</tr>
<tr>
<td>Wins</td>
<td>Losses</td>
</tr>
<tr>
<td>109</td>
<td>13</td>
</tr>
<tr>
<td>Nebraska</td>
<td>Nebraska</td>
</tr>
<tr>
<td>108</td>
<td>16</td>
</tr>
<tr>
<td>Marshall</td>
<td>Marshall</td>
</tr>
<tr>
<td>114</td>
<td>25</td>
</tr>
<tr>
<td>Florida</td>
<td>Florida</td>
</tr>
<tr>
<td>102</td>
<td>22</td>
</tr>
<tr>
<td>Tennessee</td>
<td>Tennessee</td>
</tr>
<tr>
<td>99</td>
<td>22</td>
</tr>
</tbody>
</table>

Use subtraction of matrices to determine the regular season records of these teams during the decade.
So, the regular season records of the teams can be described as follows.

<table>
<thead>
<tr>
<th>Regular Season Record</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wins</td>
</tr>
<tr>
<td>Florida State</td>
</tr>
<tr>
<td>Nebraska</td>
</tr>
<tr>
<td>Marshall</td>
</tr>
<tr>
<td>Florida</td>
</tr>
<tr>
<td>Tennessee</td>
</tr>
</tbody>
</table>

You can multiply any matrix by a constant called a scalar. This is called **scalar multiplication**. When scalar multiplication is performed, each element is multiplied by the scalar and a new matrix is formed.

### Key Concept

**Scalar Multiplication of a Matrix**

\[ m \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} ma & mb & mc \\ md & me & mf \end{bmatrix} \]

### Example 4

**Perform Scalar Multiplication**

If \( T = \begin{bmatrix} -4 & 2 \\ 0 & 1 \\ 3 & -6 \end{bmatrix} \), find \( 3T \).

\[
3T = 3 \begin{bmatrix} -4 & 2 \\ 0 & 1 \\ 3 & -6 \end{bmatrix} = \begin{bmatrix} 3(-4) & 3(2) \\ 3(0) & 3(1) \\ 3(3) & 3(-6) \end{bmatrix} = \begin{bmatrix} -12 & 6 \\ 0 & 3 \\ 9 & -18 \end{bmatrix}
\]

### Check for Understanding

**Concept Check**

1. Describe the difference between a 2-by-4 matrix and a 4-by-2 matrix.

2. **OPEN ENDED** Write two matrices whose sum is \( \begin{bmatrix} 0 & 4 & 5 & -3 \\ 1 & -1 & 4 & 9 \end{bmatrix} \).

3. **FIND THE ERROR** Hiroshi and Estrella are finding \( -5 \begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix} \).

Hiroshi:

\[
-5 \begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 10 & 5 \end{bmatrix}
\]

Estrella:

\[
-5 \begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 5 & -15 \\ 10 & -25 \end{bmatrix}
\]

Who is correct? Explain your reasoning.

[www.algebra1.com/extra_examples](http://www.algebra1.com/extra_examples)
Guided Practice

State the dimensions of each matrix. Then, identify the position of the circled element in each matrix.

4. \[
\begin{bmatrix}
4 & 0 \\
5 & -1 \\
6 & 2 \\
\end{bmatrix}
\]

5. \[
\begin{bmatrix}
3 & -3 & 1 \\
-2 & 4 & 9 \\
\end{bmatrix}
\]

6. \[
\begin{bmatrix}
5 \\
2 \\
-3 \\
\end{bmatrix}
\]

7. \[
\begin{bmatrix}
0.6 & 4.2 \\
-1.7 & 1.05 \\
0.625 & -2.1 \\
\end{bmatrix}
\]

If \(A = \begin{bmatrix} 20 & -10 \\ 12 & -19 \end{bmatrix}\), \(B = \begin{bmatrix} 15 & 14 \\ -10 & 6 \end{bmatrix}\), and \(C = \begin{bmatrix} -5 & 7 \end{bmatrix}\), find each sum, difference, or product. If the sum or difference does not exist, write impossible.

8. \(A + C\) 9. \(B - A\) 10. \(2A\) 11. \(-4C\)

Application

PIZZA SALES For Exercises 12–16, use the following tables that list the number of pizzas sold at Sylvia’s Pizza one weekend.

**FRIDAY**

<table>
<thead>
<tr>
<th></th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thin Crust</td>
<td>12</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>Thick Crust</td>
<td>11</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Deep Dish</td>
<td>14</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

**SATURDAY**

<table>
<thead>
<tr>
<th></th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thin Crust</td>
<td>13</td>
<td>12</td>
<td>11</td>
</tr>
<tr>
<td>Thick Crust</td>
<td>1</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Deep Dish</td>
<td>8</td>
<td>11</td>
<td>2</td>
</tr>
</tbody>
</table>

**SUNDAY**

<table>
<thead>
<tr>
<th></th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thin Crust</td>
<td>11</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>Thick Crust</td>
<td>1</td>
<td>8</td>
<td>11</td>
</tr>
<tr>
<td>Deep Dish</td>
<td>10</td>
<td>15</td>
<td>11</td>
</tr>
</tbody>
</table>

12. Create a matrix for each day’s data. Name the matrices \(F\), \(R\), and \(N\), respectively.

13. Does \(F\) equal \(R\)? Explain.

14. Create matrix \(T\) to represent \(F + R + N\).

15. What does \(T\) represent?

16. Which type of pizza had the most sales during the entire weekend?

Practice and Apply

State the dimensions of each matrix. Then, identify the position of the circled element in each matrix.

17. \[
\begin{bmatrix}
2 & 1 \\
5 & -8 \\
\end{bmatrix}
\]

18. \[
\begin{bmatrix}
-36 & 3 \\
25 & -1 \\
11 & 14 \\
\end{bmatrix}
\]

19. \[
\begin{bmatrix}
1 \\
0 \\
-1 \\
\end{bmatrix}
\]

20. \[
\begin{bmatrix}
-3 & 56 & -21 \\
60 & 112 & -65 \\
\end{bmatrix}
\]

21. \[
\begin{bmatrix}
-4 & 0 & -2 \\
5 & 1 & 12 \\
-6 & 3 & -7 \\
\end{bmatrix}
\]

22. \[
\begin{bmatrix}
1 & -2 \\
3 & 4 \\
1 & 5 \\
\end{bmatrix}
\]

23. \[
\begin{bmatrix}
-5 & 3 & 1 \\
4 & 0 & 2 \\
\end{bmatrix}
\]

24. \[
\begin{bmatrix}
-6 & 3 \\
5 & -4 \\
\end{bmatrix}
\]
25. Create a 2-by-3 matrix with 2 in the first row and first column and 5 in the second row and second column. The rest of the elements should be ones.

26. Create a 3-by-2 matrix with 8 in the second row and second column and 4 in the third row and second column. The rest of the elements should be zeros.

\[
\begin{bmatrix}
-1 & 5 & 9 \\
0 & -4 & -2 \\
3 & 7 & 6
\end{bmatrix}, \quad
\begin{bmatrix}
-12 & 7 & -16 \\
5 & 10 & 13 \\
20 & 11 & 8
\end{bmatrix}, \quad
\begin{bmatrix}
34 & 91 & 63 \\
81 & 79 & 60
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
-52 & 9 & 70 \\
-49 & -8 & 45
\end{bmatrix}
\]

find each sum, difference, or product. If the sum or difference does not exist, write impossible.

27. \( A + B \)

28. \( C + D \)

29. \( C - D \)

30. \( B - A \)

31. \( 5A \)

32. \( 2C \)

33. \( A + C \)

34. \( B + D \)

35. \( 2B + A \)

36. \( 4A - B \)

37. \( 2C - 3D \)

38. \( 5D + 2C \)

**FOOD** For Exercises 39–41, use the table that shows the nutritional value of food.

<table>
<thead>
<tr>
<th>Food</th>
<th>Calories</th>
<th>Protein (grams)</th>
<th>Fat (grams)</th>
<th>Saturated Fat (grams)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fish Stick</td>
<td>70</td>
<td>6</td>
<td>3</td>
<td>0.8</td>
</tr>
<tr>
<td>Vegetable Soup (1 cup)</td>
<td>70</td>
<td>2</td>
<td>2</td>
<td>0.3</td>
</tr>
<tr>
<td>Soft Drink (12 oz)</td>
<td>160</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Chocolate-Chip Cookie</td>
<td>185</td>
<td>2</td>
<td>11</td>
<td>3.9</td>
</tr>
</tbody>
</table>

Source: U.S. Department of Agriculture

39. If \( F = \begin{bmatrix} 70 & 6 & 3 & 0.8 \end{bmatrix} \) is a matrix representing the nutritional value of a fish stick, create matrices \( V, S, \) and \( C \) to represent vegetable soup, soft drink, and chocolate chip cookie, respectively.

40. Suppose Lakeisha has two fish sticks for lunch. Write a matrix representing the nutritional value of the fish sticks.

41. Suppose Lakeisha has two fish sticks, a cup of vegetable soup, a 12-ounce soft drink, and a chocolate chip cookie. Write a matrix representing the nutritional value of her lunch.

**FUND-RAISING** For Exercises 42–44, use the table that shows the last year’s sales of T-shirts for the student council fund-raiser.

<table>
<thead>
<tr>
<th>Color</th>
<th>XS</th>
<th>S</th>
<th>M</th>
<th>L</th>
<th>XL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>18</td>
<td>28</td>
<td>32</td>
<td>24</td>
<td>21</td>
</tr>
<tr>
<td>White</td>
<td>24</td>
<td>30</td>
<td>45</td>
<td>47</td>
<td>25</td>
</tr>
<tr>
<td>Blue</td>
<td>17</td>
<td>19</td>
<td>26</td>
<td>30</td>
<td>28</td>
</tr>
</tbody>
</table>

42. Create a matrix to show the number of T-shirts sold last year according to size and color. Label this matrix \( N \).

43. The student council anticipates a 20% increase in T-shirt sales this year. What value of the scalar \( r \) should be used so that \( rN \) results in a matrix that estimates the number of each size and color T-shirts needed this year?

44. Calculate \( rN \), rounding appropriately, to show estimates for this year’s sales.
FOOTBALL  For Exercises 45–48, use the table that shows the passing performance of four National Football League quarterbacks.

1999 Regular Season

<table>
<thead>
<tr>
<th>Quarterback</th>
<th>Attempts</th>
<th>Completions</th>
<th>Passing Yards</th>
<th>Touchdowns</th>
<th>Interceptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peyton Manning</td>
<td>533</td>
<td>331</td>
<td>4135</td>
<td>26</td>
<td>15</td>
</tr>
<tr>
<td>Rich Gannon</td>
<td>515</td>
<td>304</td>
<td>3840</td>
<td>24</td>
<td>14</td>
</tr>
<tr>
<td>Kurt Warner</td>
<td>499</td>
<td>325</td>
<td>4353</td>
<td>41</td>
<td>13</td>
</tr>
<tr>
<td>Steve Beuerlein</td>
<td>571</td>
<td>343</td>
<td>4436</td>
<td>36</td>
<td>15</td>
</tr>
</tbody>
</table>

2000 Regular Season

<table>
<thead>
<tr>
<th>Quarterback</th>
<th>Attempts</th>
<th>Completions</th>
<th>Passing Yards</th>
<th>Touchdowns</th>
<th>Interceptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peyton Manning</td>
<td>571</td>
<td>357</td>
<td>4413</td>
<td>33</td>
<td>15</td>
</tr>
<tr>
<td>Rich Gannon</td>
<td>473</td>
<td>284</td>
<td>3430</td>
<td>28</td>
<td>11</td>
</tr>
<tr>
<td>Kurt Warner</td>
<td>347</td>
<td>235</td>
<td>3429</td>
<td>21</td>
<td>18</td>
</tr>
<tr>
<td>Steve Beuerlein</td>
<td>533</td>
<td>324</td>
<td>3730</td>
<td>19</td>
<td>18</td>
</tr>
</tbody>
</table>

Source: ESPN

45. Create matrix $A$ for the 1999 data and matrix $B$ for the 2000 data.

46. What are the dimensions of each matrix in Exercise 45?

47. Calculate $T = A + B$.

48. What does matrix $T$ represent?

49. CRITICAL THINKING  Suppose $M$ and $N$ are each 3-by-3 matrices. Determine whether each statement is sometimes, always, or never true.

   a. $M = N$
   b. $M + N = N + M$
   c. $M - N = N - M$
   d. $5M = M$
   e. $M + N = M$
   f. $5M = N$

50. WRITING IN MATH  Answer the question that was posed at the beginning of the lesson.

**How are matrices used to organize data?**

Include the following in your answer:

- a comparison of a table and a matrix, and
- description of some real-world data that could be organized in a matrix.

51. Which of the following is equal to $\begin{bmatrix} 3 & 4 & 5 \\ -6 & -1 & 8 \end{bmatrix}$?

   A $\begin{bmatrix} -1 & 8 & 3 \\ -4 & 0 & 5 \end{bmatrix} + \begin{bmatrix} 4 & -4 & 2 \\ 2 & -1 & -2 \end{bmatrix}$
   B $\begin{bmatrix} 7 & -1 & 2 \\ 3 & 4 & -5 \end{bmatrix} + \begin{bmatrix} -4 & -3 & 3 \\ -3 & -5 & -3 \end{bmatrix}$
   C $\begin{bmatrix} 1 & -3 & 5 \\ 7 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 7 & 0 \\ -13 & 1 & 8 \end{bmatrix}$
   D $\begin{bmatrix} 5 & 9 & -2 \\ 3 & 7 & 5 \end{bmatrix} + \begin{bmatrix} -2 & -5 & -3 \\ 3 & -8 & 3 \end{bmatrix}$

52. Suppose $M$ and $N$ are each 2-by-2 matrices. If $M + N = M$, which of the following is true?

   A $N = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
   B $N = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
   C $N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
   D $N = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Getting Ready for the Next Lesson

**PREREQUISITE SKILL** For Exercises 69 and 70, use the graph that shows the amount of money in Megan’s savings account. *(To review interpreting graphs, see Lesson 1-9.)*

69. Describe what is happening to Megan’s bank balance. Give possible reasons why the graph rises and falls at particular points.

70. Describe the elements in the domain and range.

Practice Quiz 1

**Lessons 13-1 and 13-2**

Identify each sample, suggest a population from which it was selected, and state whether it is unbiased (random) or biased. If unbiased, classify the sample as simple, stratified, or systematic. If biased, classify as convenience or voluntary response. *(Lesson 13-1)*

1. Every other household in a neighborhood is surveyed to determine how to improve the neighborhood park.

2. Every other household in a neighborhood is surveyed to determine the favorite candidate for the state’s governor.

Find each sum, difference, or product. *(Lesson 13-2)*

3. \[
\begin{bmatrix}
-8 & 3 \\
-4 & -9
\end{bmatrix}
+ 
\begin{bmatrix}
5 & -7 \\
-1 & 0
\end{bmatrix}
\]

4. \[
\begin{bmatrix}
-9 & 6 & 4 \\
-1 & 3 & 2
\end{bmatrix}
- 
\begin{bmatrix}
7 & -2 & 8 \\
5 & -3 & 1
\end{bmatrix}
\]

5. \[
\begin{bmatrix}
8 & -3 & -4 \\
6 & -1 & 2
\end{bmatrix}
\]

**Mixed Review**

**PRINTING** For Exercises 58 and 59, use the following information.

To determine the quality of calendars printed at a local shop, the last 10 calendars printed each day are examined. *(Lesson 13-1)*

58. Identify the sample.

59. State whether it is unbiased (random) or biased. If unbiased, classify the sample as simple, stratified, or systematic. If biased, classify as convenience or voluntary response.

Solve each equation. *(Lesson 12-9)*

60. \[
\frac{-4}{a+1} + \frac{3}{a} = 1
\]

61. \[
\frac{3}{x} + \frac{4x}{x-3} = 4
\]

62. \[
\frac{d + 3}{d + 5} + \frac{2}{d - 9} = \frac{5}{2d + 10}
\]

Find the \(n\)th term of each geometric sequence. *(Lesson 10-7)*

63. \(a_1 = 4, n = 5, r = 3\)

64. \(a_1 = -2, n = 3, r = 7\)

65. \(a_1 = 4, n = 5, r = -2\)

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write prime. *(Lesson 9-3)*

66. \(b^2 + 7b + 12\)

67. \(a^2 + 2ab - 3b^2\)

68. \(d^2 + 8d - 15\)

**Maintain Your Skills**

**MATRIX OPERATIONS** You can use a graphing calculator to perform matrix operations. Use the EDIT command on the MATRIX menu of a TI-83 Plus to enter each of the following matrices.

\[
A = \begin{bmatrix}
-7.9 & 5.4 & -6.8 \\
-5.9 & 4.4 & -7.7
\end{bmatrix}
\quad B = \begin{bmatrix}
-7.2 & -5.8 & 9.1 \\
4.3 & -8.4 & 5.3
\end{bmatrix}
\quad C = \begin{bmatrix}
9.8 & -1.2 & 5.2 \\
-7.8 & 5.1 & -9.0
\end{bmatrix}
\]

Use these stored matrices to find each sum, difference, or product.

53. \(A + B\)

54. \(C - B\)

55. \(B + C - A\)

56. \(1.8A\)

57. \(0.4C\)
How are histograms used to display data?

A frequency table shows the frequency of events. The frequency table below shows the number of states with the mean SAT verbal and mathematics scores in each score interval. The data are from the 1999–2000 school year.

<table>
<thead>
<tr>
<th>Score Interval</th>
<th>Verbal Number of States</th>
<th>Mathematics Number of States</th>
</tr>
</thead>
<tbody>
<tr>
<td>480 ≤ s &lt; 500</td>
<td>11</td>
<td>5</td>
</tr>
<tr>
<td>500 ≤ s &lt; 520</td>
<td>10</td>
<td>18</td>
</tr>
<tr>
<td>520 ≤ s &lt; 540</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>540 ≤ s &lt; 560</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>560 ≤ s &lt; 580</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>580 ≤ s &lt; 600</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>600 ≤ s &lt; 620</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

Source: The College Board

The distribution of the mean scores on the SAT verbal exam is displayed in the graph.

### Interpret Data in Histograms

The graph above is called a histogram. A histogram is a bar graph in which the data are organized into equal intervals. In the histogram above, the horizontal axis shows the range of data values separated into measurement classes, and the vertical axis shows the number of values, or the frequency, in each class. Consider the histogram shown below.

- The width of each measurement class is 5 points.
- There are seven classes on the histogram. Note that although 45–50 has a frequency of 0, it is a measurement class.
A histogram is a visual summary of a frequency table.

**Example 1** Determine Information from a Histogram

**GEOGRAPHY** Answer each question about the histogram shown below.

a. In what measurement class does the median occur?

First, add the frequencies to determine the number of counties in Maine.

\[ 7 + 4 + 3 + 1 + 0 + 1 = 16 \]

There are 16 counties, so the middle data value is between the 8th and 9th data values. Both the 8th and 9th data values are located in the 50–100 thousand measurement class. Therefore, the median occurs in the 50–100 thousand measurement class.

b. Describe the distribution of the data.

- Only two counties have populations above 150 thousand. It is likely that these counties contain the largest cities in Maine.
- There is a gap in the 200–250 thousand measurement class.
- Most of the counties have populations below 150 thousand.
- As population increases, the histogram shows that the number of counties decreases. We say that the distribution is skewed, or pulled in one direction away from the center. This distribution is skewed to the left because the majority of the data are located at the low end of the scale.

You can sometimes use the appearances of histograms to compare data.

**Example 2** Compare Data in Histograms

**Multiple-Choice Test Item**

Which group of students has a greater median height?

\[ \text{Heights of Classmates, Group A} \]

\[ \text{Heights of Classmates, Group B} \]

\[ \text{A} \quad \text{Group A} \]

\[ \text{B} \quad \text{Group B} \]

\[ \text{C} \quad \text{The medians are about the same.} \]

\[ \text{D} \quad \text{cannot be determined} \]
Read the Test Item
You have two histograms depicting the heights of two groups of students. You are asked to determine which group of students has a greater median height.

Solve the Test Item
Study the histograms carefully. The measurement classes and the frequency scales are the same for each histogram. The distribution for Group A is somewhat symmetrical in shape, while the distribution for Group B is skewed to the right. This would indicate that Group B has the greater median height. To check this assumption, locate the measurement class of each median.

Group A

\[4 + 6 + 8 + 5 + 4 + 1 = 28\]
The median is between the 14th and 15th data values. The median is in the 140–150 measurement class.

Group B

\[2 + 3 + 5 + 6 + 8 + 7 = 31\]
The median is the 16th data value. The median is in the 150–160 measurement class.

This confirms that Group B has the greater median height. The answer is B.

**DISPLAY DATA IN A HISTOGRAM**
Data from a list or a frequency table can be used to create a histogram.

**Example 3 Create a Histogram**

**SCHOOL**
Create a histogram to represent the following scores for a 50-point mathematics test.

\[40, 34, 38, 23, 41, 39, 39, 34, 43, 44, 32, 44, 41, 39, 22, 47, 36, 25, 41, 30, 28, 37, 39, 33, 30, 40, 28\]

**Step 1** Identify the greatest and least values in the data set.
The test scores range from 22 to 47 points.

**Step 2** Create measurement classes of equal width.
For these data, use measurement classes from 20 to 50 with a 5-point interval for each class.

**Step 3** Create a frequency table using the measurement classes.

<table>
<thead>
<tr>
<th>Score Intervals</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>[20 \leq s &lt; 25]</td>
<td>II</td>
<td>2</td>
</tr>
<tr>
<td>[25 \leq s &lt; 30]</td>
<td>III</td>
<td>3</td>
</tr>
<tr>
<td>[30 \leq s &lt; 35]</td>
<td>IIII I</td>
<td>6</td>
</tr>
<tr>
<td>[35 \leq s &lt; 40]</td>
<td>IIII II</td>
<td>7</td>
</tr>
<tr>
<td>[40 \leq s &lt; 45]</td>
<td>IIII III</td>
<td>8</td>
</tr>
<tr>
<td>[45 \leq s &lt; 50]</td>
<td>I</td>
<td>1</td>
</tr>
</tbody>
</table>

**Step 4** Draw the histogram.
Use the measurement classes to determine the scale for the horizontal axis and the frequency values to determine the scale for the vertical axis. For each measurement class, draw a rectangle as wide as the measurement class and as tall as the frequency for the class. Label the axes and include a descriptive title for the histogram.
Check for Understanding

Concept Check

1. Describe how to create a histogram.
2. Write a compound inequality to represent all of the values \( v \) included in a 50–60 measurement class.
3. OPEN ENDED Write a set of data whose histogram would be skewed to the left.

Guided Practice

MONEY For Exercises 4 and 5, use the following histogram that shows the amount of money spent by several families during a holiday weekend.

4. In what measurement class does the median occur?
5. Describe the distribution of the data.

SCHOOL For Exercises 6 and 7, use the following histograms.

6. Compare the medians of the two data sets.
7. Compare and describe the overall shape of each distribution of data.
8. AIR TRAVEL The busiest U.S. airports as determined by the number of passengers arriving and departing are listed below. Create a histogram.

<table>
<thead>
<tr>
<th>Airport</th>
<th>Passengers (millions)</th>
<th>Airport</th>
<th>Passengers (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atlanta (Hartsfield)</td>
<td>80</td>
<td>Minneapolis/St. Paul</td>
<td>37</td>
</tr>
<tr>
<td>Chicago (O’Hare)</td>
<td>72</td>
<td>Phoenix (Sky Harbor)</td>
<td>36</td>
</tr>
<tr>
<td>Los Angeles</td>
<td>68</td>
<td>Detroit</td>
<td>36</td>
</tr>
<tr>
<td>Dallas/Fort Worth</td>
<td>61</td>
<td>Houston (George Bush)</td>
<td>35</td>
</tr>
<tr>
<td>San Francisco</td>
<td>41</td>
<td>Newark</td>
<td>34</td>
</tr>
<tr>
<td>Denver</td>
<td>39</td>
<td>Miami</td>
<td>34</td>
</tr>
<tr>
<td>Las Vegas (McCardan)</td>
<td>37</td>
<td>New York (JFK)</td>
<td>33</td>
</tr>
</tbody>
</table>

Source: Airports Council International

Online Research Data Update What are the current busiest airports? Visit www.algebra1.com/data_update to get statistics on airports.
9. Which statement about the graph at the right is not correct?

A. The data are skewed to the left.
B. The median is in the 40–50 thousand measurement class.
C. There are 32 employees represented by the graph.
D. The width of each measurement class is $10 thousand.

For each histogram, answer the following.

• In what measurement class does the median occur?
• Describe the distribution of the data.

10. 

11. 

For each pair of histograms, answer the following.

• Compare the medians of the two data sets.
• Compare and describe the overall shape of each distribution of data.

12. 

13.
Create a histogram to represent each data set.

14. Students’ semester averages in a mathematics class: 96.53, 95.96, 94.25, 93.58, 91.91, 90.33, 90.27, 90.11, 89.30, 89.06, 88.33, 88.30, 87.43, 86.67, 86.31, 84.21, 83.53, 82.30, 78.71, 77.51, 73.83

15. Number of raisins found in a snack-size box: 54, 59, 55, 109, 97, 59, 102, 68, 104, 63, 101, 59, 96, 58, 57, 63, 94, 61, 104, 62, 58, 59, 102, 60, 54, 58, 53, 78

**BASEBALL** For Exercises 16 and 17, use the following table.

<table>
<thead>
<tr>
<th>Team</th>
<th>Payroll (millions)</th>
<th>Team</th>
<th>Payroll (millions)</th>
<th>Team</th>
<th>Payroll (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yankees</td>
<td>$112</td>
<td>Orioles</td>
<td>$59</td>
<td>White Sox</td>
<td>$37</td>
</tr>
<tr>
<td>Braves</td>
<td>$94</td>
<td>Tigers</td>
<td>$59</td>
<td>Reds</td>
<td>$36</td>
</tr>
<tr>
<td>Red Sox</td>
<td>$91</td>
<td>Rockies</td>
<td>$56</td>
<td>Phillies</td>
<td>$36</td>
</tr>
<tr>
<td>Dodgers</td>
<td>$90</td>
<td>Padres</td>
<td>$55</td>
<td>Athletics</td>
<td>$32</td>
</tr>
<tr>
<td>Mets</td>
<td>$82</td>
<td>Blue Jays</td>
<td>$54</td>
<td>Pirates</td>
<td>$29</td>
</tr>
<tr>
<td>Indians</td>
<td>$77</td>
<td>Giants</td>
<td>$54</td>
<td>Expos</td>
<td>$28</td>
</tr>
<tr>
<td>Diamondbacks</td>
<td>$74</td>
<td>Angels</td>
<td>$53</td>
<td>Brewers</td>
<td>$26</td>
</tr>
<tr>
<td>Cardinals</td>
<td>$73</td>
<td>Devil Rays</td>
<td>$51</td>
<td>Marlins</td>
<td>$25</td>
</tr>
<tr>
<td>Rangers</td>
<td>$62</td>
<td>Astros</td>
<td>$51</td>
<td>Royals</td>
<td>$24</td>
</tr>
<tr>
<td>Mariners</td>
<td>$62</td>
<td>Cubs</td>
<td>$50</td>
<td>Twins</td>
<td>$15</td>
</tr>
</tbody>
</table>

Source: USA TODAY

16. Create a histogram to represent the payroll data.

17. On your histogram, locate and label the median team payroll.

**ELECTIONS** For Exercises 18–20, use the following table.

<table>
<thead>
<tr>
<th>State</th>
<th>Percent</th>
<th>State</th>
<th>Percent</th>
<th>State</th>
<th>Percent</th>
<th>State</th>
<th>Percent</th>
<th>State</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>MN</td>
<td>68.75</td>
<td>WY</td>
<td>59.70</td>
<td>OH</td>
<td>55.76</td>
<td>MD</td>
<td>51.56</td>
<td>AR</td>
<td>47.79</td>
</tr>
<tr>
<td>ME</td>
<td>67.34</td>
<td>CT</td>
<td>58.40</td>
<td>ID</td>
<td>54.46</td>
<td>NJ</td>
<td>51.04</td>
<td>NM</td>
<td>47.40</td>
</tr>
<tr>
<td>AK</td>
<td>66.41</td>
<td>SD</td>
<td>58.24</td>
<td>RI</td>
<td>54.29</td>
<td>FL</td>
<td>50.65</td>
<td>SC</td>
<td>46.49</td>
</tr>
<tr>
<td>WI</td>
<td>66.07</td>
<td>MI</td>
<td>57.52</td>
<td>LA</td>
<td>54.24</td>
<td>NC</td>
<td>50.28</td>
<td>WV</td>
<td>45.74</td>
</tr>
<tr>
<td>VT</td>
<td>63.98</td>
<td>MO</td>
<td>57.49</td>
<td>KS</td>
<td>54.07</td>
<td>AL</td>
<td>49.99</td>
<td>CA</td>
<td>44.09</td>
</tr>
<tr>
<td>NH</td>
<td>62.33</td>
<td>WA</td>
<td>56.95</td>
<td>PA</td>
<td>53.66</td>
<td>IN</td>
<td>49.44</td>
<td>GA</td>
<td>43.84</td>
</tr>
<tr>
<td>MT</td>
<td>61.52</td>
<td>MA</td>
<td>56.92</td>
<td>IL</td>
<td>52.79</td>
<td>NY</td>
<td>49.42</td>
<td>NV</td>
<td>43.81</td>
</tr>
<tr>
<td>IA</td>
<td>60.71</td>
<td>CO</td>
<td>56.78</td>
<td>UT</td>
<td>52.61</td>
<td>TN</td>
<td>49.19</td>
<td>TX</td>
<td>43.15</td>
</tr>
<tr>
<td>OR</td>
<td>60.63</td>
<td>NE</td>
<td>56.44</td>
<td>VA</td>
<td>52.05</td>
<td>OK</td>
<td>48.76</td>
<td>AZ</td>
<td>42.26</td>
</tr>
<tr>
<td>ND</td>
<td>60.63</td>
<td>DE</td>
<td>56.22</td>
<td>KY</td>
<td>51.59</td>
<td>MS</td>
<td>48.57</td>
<td>HI</td>
<td>40.48</td>
</tr>
</tbody>
</table>

Source: USA TODAY

18. Determine the median of the data.

19. Create a histogram to represent the data.

20. Write a sentence or two describing the distribution of the data.

21. **RESEARCH** Choose your favorite professional sport. Use the Internet or other reference to find how many games each team in the appropriate league won last season. Use this information to create a histogram. Describe your histogram.

22. **CRITICAL THINKING** Create a histogram with a gap between 20 and 40, one item in the 50–55 measurement class, and the median value in the 50–55 measurement class.

www.algebra1.com/self_check_quiz
23. **Writing in Math**  Answer the question that was posed at the beginning of the lesson.

How are histograms used to display data?
Include the following in your answer:
- the advantage of the histogram over the frequency table, and
- a histogram depicting the distribution of the mean scores on the SAT mathematics exam.

For Exercises 24 and 25, use the information in the graph.

24. How many employees are represented in the graph?
   - **A** 38  
   - **B** 40  
   - **C** 46  
   - **D** 48

25. In which measurement class is the median of the data located?
   - **A** 2–4  
   - **B** 4–6  
   - **C** 6–8  
   - **D** 8–10

**Histograms**  You can use a graphing calculator to create histograms. On a TI-83 Plus, enter the data in L1. In the STAT PLOT menu, turn on Plot 1 and select the histogram. Define the viewing window and press **[GRAPH]**.

Use a graphing calculator to create a histogram for each set of data.

26. 5, 5, 6, 7, 9, 4, 10, 12, 13, 8, 15, 16, 13, 8
27. 12, 14, 25, 30, 11, 35, 41, 47, 13, 18, 58, 59, 42, 13, 18
28. 124, 83, 81, 110, 92, 178, 179, 134, 92, 133, 145, 180, 144
29. 2.2, 2.4, 7.5, 9.1, 3.4, 5.1, 6.3, 1.8, 2.8, 3.7, 8.6, 9.5, 3.6, 3.7, 5.0

**Maintain Your Skills**

**Mixed Review**

If \( A = \begin{bmatrix} -2 & 3 & 7 \\ 0 & -4 & 6 \\ 1 & -5 & 4 \end{bmatrix} \), \( B = \begin{bmatrix} -8 & 1 & -1 \\ 2 & 3 & -7 \end{bmatrix} \), and \( C = \begin{bmatrix} 7 & -5 & 2 \\ 0 & 0 & 3 \\ -1 & 4 & 6 \end{bmatrix} \), find each sum, difference, or product. If the sum or difference does not exist, write impossible.  \( \text{(Lesson 13-2)} \)

30. \( A + B \)  
31. \( C - A \)  
32. \( 2B \)  
33. \( -5A \)

34. **Manufacturing**  Every 15 minutes, a CD player is taken off the assembly line and tested. State whether this sample is unbiased (random) or biased. If unbiased, classify the sample as simple, stratified, or systematic. If biased, classify as convenience or voluntary response.  \( \text{(Lesson 13-1)} \)

Find each quotient. Assume that no denominator has a value of 0.  \( \text{(Lesson 12-4)} \)

35. \( \frac{s}{s + 7} \div \frac{s - 5}{s + 7} \)  
36. \( \frac{2m^2 + 7m - 15}{m + 2} \div \frac{2m - 3}{m^2 + 5m + 6} \)

Solve each equation. Check your solution.  \( \text{(Lesson 11-3)} \)

37. \( \sqrt{y} + 3 + 5 = 9 \)  
38. \( \sqrt{x - 2} = x - 4 \)  
39. \( 13 = \sqrt{2w - 5} \)

**Getting Ready for the Next Lesson**

**Prerequisite Skill**  Find the median for each set of data.  \( \text{(To review median, see pages 818 and 819.)} \)

40. 2, 4, 7, 9, 12, 15
41. 10, 3, 17, 1, 8, 6, 12, 15
42. 7, 19, 9, 4, 7, 2
43. 2.1, 7.4, 13.9, 1.6, 5.21, 3.901
Graphing Calculator Investigation

Curve Fitting

If there is a constant increase or decrease in data values, there is a linear trend. If the values are increasing or decreasing more and more rapidly, there may be a quadratic or exponential trend. The curvature of a quadratic trend tends to appear more gradual. Below are three scatter plots, each showing a different trend.

With a TI-83 Plus, you can use the LinReg, QuadReg, and ExpReg functions to find the appropriate regression equation that best fits the data.

**FARMING** A study is conducted in which groups of 25 corn plants are given a different amount of fertilizer and the gain in height after a certain time is recorded. The table below shows the results.

<table>
<thead>
<tr>
<th>Fertilizer (mg)</th>
<th>0</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain in Height (in.)</td>
<td>6.48</td>
<td>7.35</td>
<td>8.73</td>
<td>9.00</td>
<td>8.13</td>
</tr>
</tbody>
</table>

**Step 1** Make a scatter plot.
- Enter the fertilizer in L1 and the height in L2.
  **KEYSTROKES:** Review entering a list on page 204.
- Use STAT PLOT to graph the scatter plot.
  **KEYSTROKES:** Review statistical plots on page 204. Use ZOOM 9 to graph.

The graph appears to be a quadratic regression.

**Step 2** Find the quadratic regression equation.
- Select QuadReg on the STAT CALC menu.
  **KEYSTROKES:** STAT → 5 ENTER

The equation is about $y = -0.0008x^2 + 0.1x + 6.3$.

$R^2$ is the coefficient of determination. The closer $R^2$ is to 1, the better the model. To choose a quadratic or exponential model, fit both and use the one with the $R^2$ value closer to 1.

(continued on the next page)
Graphing Calculator Investigation

**Step 3** Graph the quadratic regression equation.
- Copy the equation to the \( Y = \) list and graph.

**KEYSTROKES:**
- \( Y = \)
- \( \text{VARS} \)
- 5
- \( \text{GRAPH} \)
- \( \text{ZOOM} \)

**Step 4** Predict using the equation.
- Find the amount of fertilizer that produces the maximum gain in height.

On average, about 55 milligrams of the fertilizer produces the maximum gain.

**Exercises**

Plot each set of data points. Determine whether to use a *linear*, *quadratic*, or *exponential* regression equation. State the coefficient of determination.

1. \[
\begin{array}{c|c}
 x & y \\
-0.0 & 2.98 \\
0.2 & 1.46 \\
0.4 & 0.90 \\
0.6 & 0.51 \\
0.8 & 0.25 \\
1.0 & 0.13 \\
\end{array}
\]

2. \[
\begin{array}{c|c}
 x & y \\
1 & 25.9 \\
2 & 22.2 \\
3 & 20.0 \\
4 & 19.3 \\
5 & 18.2 \\
6 & 15.9 \\
\end{array}
\]

3. \[
\begin{array}{c|c}
 x & y \\
10 & 35 \\
20 & 50 \\
30 & 70 \\
40 & 88 \\
50 & 101 \\
60 & 120 \\
\end{array}
\]

4. \[
\begin{array}{c|c}
 x & y \\
1 & 3.67 \\
2 & 5.33 \\
3 & 6.33 \\
5 & 6.67 \\
7 & 4.33 \\
9 & 2.67 \\
11 & 0.78 \\
\end{array}
\]

**TECHNOLOGY** The cost of cellular phone use is expected to decrease. For Exercises 5–9, use the graph at the right.

5. Make a scatter plot of the data.

6. Find an appropriate regression equation, and state the coefficient of determination.

7. Use the regression equation to predict the expected cost in 2004.

8. Do you believe that your regression equation is appropriate for a year beyond the range of data, such as 2020? Explain.

9. What model may be more appropriate for predicting cost beyond 2003?

---

**USA TODAY Snapshots®**

**Cheaper wireless talk**

Cheaper digital networks and more competition are expected to cut the cost of wireless phone use. Per-minute average in 1998 and projected cost in the next five years:

- 1998: 33¢
- 1999: 28¢
- 2000: 25¢
- 2001: 23¢
- 2002: 22¢
- 2003: 20¢

Source: The Strategis Group

By Anne R. Carey and Marcy E. Mullins, USA TODAY
Measures of Variation

What You’ll Learn

- Find the range of a set of data.
- Find the quartiles and interquartile range of a set of data.

Vocabulary

- range
- measures of variation
- quartiles
- lower quartile
- upper quartile
- interquartile range
- outlier

How is variation used in weather?

The average monthly temperatures for three U.S. cities are given. Which city shows the greatest change in monthly highs?

To answer this question, find the difference between the greatest and least values in each data set.

Buffalo: \(80.2 - 30.2 = 50.0\)
Honolulu: \(88.7 - 80.1 = 8.6\)
Tampa: \(90.2 - 69.8 = 20.4\)

Buffalo shows the greatest change.

RANGE The difference between the greatest and the least monthly high temperatures is called the range of the temperatures.

Definition of Range

The range of a set of data is the difference between the greatest and the least values of the set.

The mean, median, and mode describe the central tendency of a set of data. The range of a set of data is a measure of the spread of the data. Measures that describe the spread of the values in a set of data are called measures of variation.

Example 1 Find the Range

HOCKEY The number of wins for each team in the Eastern Conference of the NHL for the 1999–2000 season are listed below. Find the range of the data.

<table>
<thead>
<tr>
<th>Team</th>
<th>Wins</th>
<th>Team</th>
<th>Wins</th>
<th>Team</th>
<th>Wins</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atlanta</td>
<td>14</td>
<td>Montreal</td>
<td>35</td>
<td>Philadelphia</td>
<td>45</td>
</tr>
<tr>
<td>Boston</td>
<td>24</td>
<td>New Jersey</td>
<td>45</td>
<td>Pittsburgh</td>
<td>37</td>
</tr>
<tr>
<td>Buffalo</td>
<td>35</td>
<td>N.Y. Islanders</td>
<td>24</td>
<td>Tampa Bay</td>
<td>19</td>
</tr>
<tr>
<td>Carolina</td>
<td>37</td>
<td>N.Y. Rangers</td>
<td>29</td>
<td>Toronto</td>
<td>45</td>
</tr>
<tr>
<td>Florida</td>
<td>43</td>
<td>Ottawa</td>
<td>41</td>
<td>Washington</td>
<td>44</td>
</tr>
</tbody>
</table>

The greatest number of wins is 45, and the least number of wins is 14. Since \(45 - 14 = 31\), the range of the number of wins is 31.
**Quartiles and Interquartile Range** In a set of data, the *quartiles* are values that separate the data into four equal subsets, each containing one fourth of the data. Statisticians often use $Q_1$, $Q_2$, and $Q_3$ to represent the three quartiles. Remember that the median separates the data into two equal parts. $Q_2$ is the median. $Q_1$ is the *lower quartile*. It divides the lower half of the data into two equal parts. Likewise $Q_3$ is the *upper quartile*. It divides the upper half of the data into two equal parts. The difference between the upper and lower quartiles is the *interquartile range* (IQR).

**Example 2** Find the Quartiles and the Interquartile Range

**Geography** The areas of the original 13 states are listed in the table. Find the median, the lower quartile, the upper quartile, and the interquartile range of the areas.

**Explore** You are given a table with the areas of the original 13 states. You are asked to find the median, the lower quartile, the upper quartile, and the interquartile range.

**Plan** First, list the areas from least to greatest. Then find the median of the data. The median will divide the data into two sets of data. To find the upper and lower quartiles, find the median of each of these sets of data. Finally, subtract the lower quartile from the upper quartile to find the interquartile range.

**Solve**

\[
\begin{align*}
Q_1 &= \frac{6 + 9}{2} = 7.5 \\
Q_3 &= \frac{46 + 54}{2} = 50
\end{align*}
\]

The median is 12 thousand square miles.

The lower quartile is 7.5 thousand square miles, and the upper quartile is 50 thousand square miles.

The interquartile range is $50 - 7.5 = 42.5$ thousand square miles.
In a set of data, a value that is much less or much greater than the rest of the data is called an outlier. An outlier is defined as any element of a set of data that is at least 1.5 interquartile ranges less than the lower quartile or greater than the upper quartile.

\[
\text{IQR} = 13 - 9 = 4
\]

\[
1 \quad 8 \quad 9 \quad 10 \\
10 \quad 11 \quad 12 \quad 13
\]

\[
13 + 1.5(4) = 19
\]

\[
9 - 1.5(4) = 3
\]

\[
23 \quad 27
\]

Example 3: Identify Outliers

Identify any outliers in the following set of data.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 \ 2 \ 7</td>
</tr>
<tr>
<td>2</td>
<td>3 \ 3 \ 3 \ 4 \ 4 \ 5 \ 6</td>
</tr>
<tr>
<td>3</td>
<td>0 \ 1 \ 4 \ 6</td>
</tr>
<tr>
<td>4</td>
<td>0 \ 6</td>
</tr>
</tbody>
</table>

Step 1: Find the quartiles.

The brackets group the values in the lower half and the values in the upper half. The boxes are used to find the lower quartile and the upper quartile.

\[
Q_1 = \frac{23 + 23}{2} \quad \text{or} \quad 23
\]

\[
Q_3 = \frac{30 + 31}{2} \quad \text{or} \quad 30.5
\]

Step 2: Find the interquartile range.

The interquartile range is 30.5 – 23 or 7.5.

Step 3: Find the outliers, if any.

An outlier must be 1.5(7.5) less than the lower quartile, 23, or 1.5(7.5) greater than the upper quartile, 30.5.

\[
23 - 1.5(7.5) = 11.75 \quad \text{or} \quad 30.5 + 1.5(7.5) = 41.75
\]

There are no values less than 11.75. Since 46 > 41.75, 46 is the only outlier.
Guided Practice

Find the range, median, lower quartile, upper quartile, and interquartile range of each set of data. Identify any outliers.

4. 85, 77, 58, 69, 62, 73, 25, 82, 67, 77, 75, 69, 76

5. Stem | Leaf
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>3 7 8</td>
</tr>
<tr>
<td>8</td>
<td>0 0 3 5 7</td>
</tr>
<tr>
<td>9</td>
<td>4 6 8</td>
</tr>
<tr>
<td>10</td>
<td>0 1 8</td>
</tr>
<tr>
<td>11</td>
<td>1 9 7</td>
</tr>
</tbody>
</table>

Application

LITTLE LEAGUE For Exercises 6–10, use the following information.
The number of runs scored by the winning team in the Little League World Series each year from 1947 to 2000 are given in the line plot below.

6. What is the range of the data? 
7. What is the median of the data? 
8. What is the lower quartile and upper quartile of the data? 
9. What is the interquartile range of the data? 
10. Name any outliers.

Practice and Apply

Find the range, median, lower quartile, upper quartile, and interquartile range of each set of data. Identify any outliers.

11. 85, 77, 58, 69, 62, 73, 55, 82, 67, 77, 75, 92, 76

12. 28, 42, 37, 31, 34, 29, 44, 28, 38, 40, 39, 42, 30

13. 30.8, 29.9, 30.0, 31.0, 30.1, 30.5, 30.7, 31.0

14. 2, 3.4, 5.3, 3, 1, 3.2, 4.9, 2.3

15. Stem | Leaf
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3 6 8</td>
</tr>
<tr>
<td>6</td>
<td>5 8</td>
</tr>
<tr>
<td>7</td>
<td>0 3 7 7 9</td>
</tr>
<tr>
<td>8</td>
<td>1 4 8 8 9</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
</tr>
</tbody>
</table>

16. Stem | Leaf
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>3 5 5</td>
</tr>
<tr>
<td>20</td>
<td>2 2 5 8</td>
</tr>
<tr>
<td>21</td>
<td>5 8 8 9 9 9</td>
</tr>
<tr>
<td>22</td>
<td>0 1 7 8 9</td>
</tr>
<tr>
<td>23</td>
<td>2</td>
</tr>
</tbody>
</table>

17. Stem | Leaf
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0 3 7 9</td>
</tr>
<tr>
<td>6</td>
<td>1 3 4 5 5 6</td>
</tr>
<tr>
<td>7</td>
<td>1 5 6 6 9</td>
</tr>
<tr>
<td>8</td>
<td>1 2 3 5 8</td>
</tr>
<tr>
<td>9</td>
<td>2 5 6 9</td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>7</td>
</tr>
</tbody>
</table>

18. Stem | Leaf
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 2 3</td>
</tr>
<tr>
<td>1</td>
<td>1 7 9</td>
</tr>
<tr>
<td>2</td>
<td>2 3 5 6</td>
</tr>
<tr>
<td>3</td>
<td>3 4 4 5 9</td>
</tr>
<tr>
<td>4</td>
<td>0 7 8 8</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>
NATIONAL PARKS  For Exercises 19–23, use the graph at the right.
19. What is the range of the visitors per month?
20. What is the median number of visitors per month?
21. What are the lower quartile and the upper quartile of the data?
22. What is the interquartile range of the data?
23. Name any outliers.

NUTRITION  For Exercises 24–28, use the following table.

<table>
<thead>
<tr>
<th>Vegetable</th>
<th>Calories</th>
<th>Vegetable</th>
<th>Calories</th>
<th>Vegetable</th>
<th>Calories</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asparagus</td>
<td>14</td>
<td>Carrots</td>
<td>28</td>
<td>Lettuce</td>
<td>9</td>
</tr>
<tr>
<td>Avocado</td>
<td>304</td>
<td>Cauliflower</td>
<td>10</td>
<td>Onion</td>
<td>60</td>
</tr>
<tr>
<td>Bell pepper</td>
<td>20</td>
<td>Celery</td>
<td>17</td>
<td>Potato</td>
<td>89</td>
</tr>
<tr>
<td>Broccoli</td>
<td>25</td>
<td>Corn</td>
<td>66</td>
<td>Spinach</td>
<td>9</td>
</tr>
<tr>
<td>Brussels sprouts</td>
<td>60</td>
<td>Green beans</td>
<td>30</td>
<td>Tomato</td>
<td>35</td>
</tr>
<tr>
<td>Cabbage</td>
<td>17</td>
<td>Jalapeno peppers</td>
<td>13</td>
<td>Zucchini</td>
<td>17</td>
</tr>
</tbody>
</table>

Source: Vitality
24. What is the range of the data?  25. What is the median of the data?
26. What are the lower quartile and the upper quartile of the data?
27. What is the interquartile range of the data?
28. Identify any outliers.

BRIDGES  For Exercises 29–33, use the following information and the double stem-and-leaf plot at the right.
The main span of cable-stayed bridges and of steel-arch bridges in the United States are given in the stem-and-leaf plot.
29. Find the ranges for each type of bridge.
30. Find the quartiles for each type of bridge.
31. Find the interquartile ranges for each type of bridge.
32. Identify any outliers.
33. Compare the ranges and interquartile ranges of the two types of bridges. What can you conclude from these statistics?

Source: The World Almanac.
34. **CRITICAL THINKING**  Trey measured the length of each classroom in his school. He then calculated the range, median, lower quartile, upper quartile, and interquartile range of the data. After his calculations, he discovered that the tape measure he had used started at the 2-inch mark instead of at the 0-inch mark. All of his measurements were 2 inches greater than the actual lengths of the rooms. How will the values that Trey calculated change? Explain your reasoning.

35. **WRITING IN MATH**  Answer the question that was posed at the beginning of the lesson.

**How is variation used in weather?**

Include the following in your answer:

- the meaning of the range and interquartile range of temperatures for a city, and
- the average highs for your community with the appropriate measures of variation.

36. What is the range of the following set of data?

\[ 53, 57, 62, 48, 45, 65, 40, 42, 55 \]

A. 11  B. 25  C. 53  D. 65

37. What is the median of the following set of data?

\[ 7, 8, 14, 3, 2, 1, 24, 18, 9, 15 \]

A. 8.5  B. 10.1  C. 11.5  D. 23

**Maintain Your Skills**

38. Create a histogram to represent the following data.  \( 36, 43, 61, 45, 37, 41, 32, 46, 60, 38, 35, 64, 46, 47, 30, 38, 48, 39 \)

39. \[ \begin{bmatrix} 5 & -3 & 6 \\ 3 & 1 & 2 \\ 4 & 9 & 3 \end{bmatrix} \]

40. \[ \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix} \]

41. \[ \begin{bmatrix} 4 & 2 & -1 & 3 \\ 5 & 9 & 0 & 2 \end{bmatrix} \]

State the dimensions of each matrix. Then identify the position of the circled element in each matrix.  \( \text{Lesson 13-2} \)

42. \[ \frac{15a}{39a^2} \]

43. \[ \frac{t-3}{t^2-7t+12} \]

44. \[ \frac{m-3}{m^2-9} \]

Simplify each rational expression. State the excluded values of the variables.  \( \text{Lesson 12-2} \)

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL**  Graph each set of numbers on a number line.

45. \{4, 7, 8, 10, 11\}  46. \{13, 17, 22, 23, 27\}  47. \{30, 35, 40, 50, 55\}

For Exercises 1–2, use the histogram at the right.  \( \text{Lesson 13-3} \)

1. In what measurement class does the median occur?

2. Describe the distribution of the data.

For Exercises 3–5, use the following set of data.  \( \text{Lesson 13-4} \)

\[ 1050, 1175, 835, 1075, 1025, 1145, 1100, 1125, 975, 1005, 1125, 1095, 1075, 1055 \]

3. Find the range of the data.

4. Find the median, the lower quartile, the upper quartile, and interquartile range of the data.

5. Identify any outliers of the data.
What You’ll Learn

• Organize and use data in box-and-whisker plots.
• Organize and use data in parallel box-and-whisker plots.

Vocabulary
• box-and-whisker plot
• extreme values

How are box-and-whisker plots used to display data?

Everyone should eat a number of calcium-rich foods each day. Selected foods and the amount of calcium in a serving are listed in the table. To create a box-and-whisker plot of the data, you need to find the quartiles of the data.

<table>
<thead>
<tr>
<th>Food</th>
<th>Calcium (milligrams)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plain Yogurt, Nonfat (8 oz)</td>
<td>452</td>
</tr>
<tr>
<td>Plain Yogurt, Low-fat (8 oz)</td>
<td>415</td>
</tr>
<tr>
<td>Skim Milk (8 oz)</td>
<td>302</td>
</tr>
<tr>
<td>1% Milk (8 oz)</td>
<td>300</td>
</tr>
<tr>
<td>Whole Milk (8 oz)</td>
<td>291</td>
</tr>
<tr>
<td>Swiss Cheese (1 oz)</td>
<td>272</td>
</tr>
<tr>
<td>Tofu (4 oz)</td>
<td>258</td>
</tr>
<tr>
<td>Sardines (2 oz)</td>
<td>217</td>
</tr>
<tr>
<td>Cheddar Cheese (1 oz)</td>
<td>204</td>
</tr>
<tr>
<td>Collards (4 oz)</td>
<td>179</td>
</tr>
<tr>
<td>American Cheese (1 oz)</td>
<td>163</td>
</tr>
<tr>
<td>Frozen Yogurt with Fruit (4 oz)</td>
<td>154</td>
</tr>
<tr>
<td>Salmon (2 oz)</td>
<td>122</td>
</tr>
<tr>
<td>Broccoli (4 oz)</td>
<td>89</td>
</tr>
</tbody>
</table>

Source: Vitality

Selected foods and the amount of calcium in a serving are listed in the table. To create a box-and-whisker plot of the data, you need to find the quartiles of the data.

89 122 154 163 179 204 217 258 272 291 300 302 415 452

\[ Q_1 \] \[ Q_2 \] \[ Q_3 \]

\[ Q_2 = \frac{217 + 258}{2} \text{ or } 237.5 \]

This information can be displayed on a number line as shown below.

BOX-AND-WHISKER PLOTS

Diagrams such as the one above are called box-and-whisker plots. The length of the box represents the interquartile range. The line inside the box represents the median. The lines or whiskers represent the values in the lower fourth of the data and the upper fourth of the data. The bullets at each end are the extreme values. In the box-and-whisker plot above, the least value (LV) is 89, and the greatest value (GV) is 452.

If a set of data has outliers, these data points are represented by bullets. The whisker representing the lower data is drawn from the box to the least value that is not an outlier. The whisker representing the upper data is drawn from the box to the greatest value that is not an outlier.
Example 1 \textbf{Draw a Box-and-Whisker Plot}

\textbf{ECOLOGY} \quad The amount of rain in Florida from January to May is crucial to its ecosystems. The following is a list of the number of inches of rain in Florida during this crucial period for the years 1990 to 2000.

14.03, 30.11, 16.03, 19.61, 18.15, 16.34, 20.43, 18.46, 22.24, 12.70, 8.25

a. Draw a box-and-whisker plot for these data.

\textbf{Step 1 \quad Determine the quartiles and any outliers.}

Order the data from least to greatest. Use this list to determine the quartiles.

\begin{align*}
8.25, \ 12.70, \ 14.03, \ 16.03, \ 16.34, \ 18.15, \ 18.46, \ 19.61, \ 20.43, \ 22.24, \ 30.11
\end{align*}

\begin{align*}
\uparrow & \quad \uparrow & \quad \uparrow \\
Q_1 & \quad Q_2 & \quad Q_3
\end{align*}

Determine the interquartile range.

\[ IQR = 20.43 - 14.03 = 6.4 \]

Check to see if there are any outliers.

\begin{align*}
14.03 - 1.5(6.4) & = 4.43 & 20.43 + 1.5(6.4) & = 30.03
\end{align*}

Any numbers less than 4.43 or greater than 30.03 are outliers. The only outlier is 30.11.

\textbf{Step 2 \quad Draw a number line.}

Assign a scale to the number line that includes the extreme values.

Above the number line, place bullets to represent the three quartile points, any outliers, the least number that is not an outlier, and the greatest number that is not an outlier.

\begin{align*}
8.25 & \quad 14.03 & \quad 18.15 & \quad 20.43 & \quad 22.24 & \quad 30.11
\end{align*}

\begin{align*}
\circ & \quad \bullet & \quad \bullet & \quad \bullet & \quad \bullet & \quad \bullet
\end{align*}

\begin{align*}
8 \quad 10 \quad 12 \quad 14 \quad 16 \quad 18 \quad 20 \quad 22 \quad 24 \quad 26 \quad 28 \quad 30 \quad 32
\end{align*}

\textbf{Step 3 \quad Complete the box-and-whisker plot.}

Draw a box to designate the data between the upper and lower quartiles. Draw a vertical line through the point representing the median. Draw a line from the lower quartile to the least value that is not an outlier. Draw a line from the upper quartile to the greatest value that is not an outlier.

\begin{align*}
\circ & \quad \bullet & \quad \bullet & \quad \bullet & \quad \bullet & \quad \bullet
\end{align*}

\begin{align*}
8 \quad 10 \quad 12 \quad 14 \quad 16 \quad 18 \quad 20 \quad 22 \quad 24 \quad 26 \quad 28 \quad 30 \quad 32
\end{align*}

b. What does the box-and-whisker plot tell about the data?

Notice that the whisker and the box for the top half of the data is shorter than the whisker and box for the lower half of the data. Therefore, except for the outlier, the upper half of the data are less spread out than the lower half of the data.

\textbf{PARALLEL BOX-AND-WHISKER PLOTS} \quad Two sets of data can be compared by drawing parallel box-and-whisker plots such as the one shown below.

\begin{align*}
\text{Data A} & \quad \bullet & \quad \bullet & \quad \bullet & \quad \bullet & \quad \bullet
\end{align*}

\begin{align*}
\text{Data B} & \quad \bullet & \quad \bullet & \quad \bullet & \quad \bullet & \quad \bullet
\end{align*}
WEATHER
Jalisa Thompson has job offers in Fresno, California, and Brownsville, Texas. Since she likes both job offers, she decides to compare the temperatures of each city.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Fresno</td>
<td>54.1</td>
<td>61.7</td>
<td>66.6</td>
<td>75.1</td>
<td>84.2</td>
<td>92.7</td>
<td>98.6</td>
<td>96.7</td>
<td>90.1</td>
<td>79.7</td>
<td>64.7</td>
<td>53.7</td>
</tr>
<tr>
<td>Brownsville</td>
<td>68.9</td>
<td>72.2</td>
<td>78.4</td>
<td>84.0</td>
<td>87.8</td>
<td>91.0</td>
<td>93.3</td>
<td>93.6</td>
<td>90.4</td>
<td>85.3</td>
<td>78.3</td>
<td>71.7</td>
</tr>
</tbody>
</table>

Source: www.stormfax.com

a. Draw a parallel box-and-whisker plot for the data.
Determine the quartiles and outliers for each city.

Fresno
53.7, 54.1, 61.7, 64.7, 66.6, 75.1, 79.7, 84.2, 90.1, 92.7, 96.7, 98.6

\[ Q_1 = 63.2 \quad Q_2 = 77.4 \quad Q_3 = 91.4 \]

Brownsville
68.9, 71.7, 72.2, 78.3, 78.4, 84.0, 85.3, 87.8, 90.4, 91.0, 93.3, 93.6

\[ Q_1 = 75.25 \quad Q_2 = 84.65 \quad Q_3 = 90.7 \]

Neither city has any outliers.

Draw the box-and-whisker plots using the same number line.

b. Use the parallel box-and-whisker plots to compare the data.
The range of temperatures in Fresno is much greater than in Brownsville.
Except for the fourth quartile, Brownville’s average temperatures appear to be as high or higher than Fresno’s.

**Check for Understanding**

**Concept Check**

1. Describe the data represented by the box-and-whisker plot at the right. Include the extreme values, the quartiles, and any outliers.

2. Explain how to determine the scale of the number line in a box-and-whisker plot.

3. OPEN ENDED Write a set of data that could be represented by the box-and-whisker plot at the right.

**Guided Practice**

Draw a box-and-whisker plot for each set of data.

4. 30, 28, 24, 24, 22, 22, 21, 17, 16, 15

5. 64, 69, 65, 71, 66, 66, 74, 67, 68, 67

www.algebra1.com/extra_examples
Draw a parallel box-and-whisker plot for each pair of data. Compare the data.

6. A: 22, 18, 22, 17, 32, 24, 31, 26, 28
   B: 28, 30, 45, 23, 24, 32, 30, 27, 27

7. A: 8, 15.5, 14, 14, 24, 19, 16.7, 15
   B: 18, 14, 15.8, 9, 12, 16, 20, 16, 13, 15

Application

CHARITY  For Exercises 8 and 9, use the information in the table below.

<table>
<thead>
<tr>
<th>Charity</th>
<th>Private Contributions (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salvation Army</td>
<td>$1397</td>
</tr>
<tr>
<td>YMCA of the U.S.A.</td>
<td>$693</td>
</tr>
<tr>
<td>American Red Cross</td>
<td>$678</td>
</tr>
<tr>
<td>American Cancer Society</td>
<td>$620</td>
</tr>
<tr>
<td>Fidelity Investments Charitable Gift Fund</td>
<td>$573</td>
</tr>
<tr>
<td>Lutheran Services in America</td>
<td>$559</td>
</tr>
<tr>
<td>United Jewish Communities</td>
<td>$524</td>
</tr>
<tr>
<td>America’s Second Harvest</td>
<td>$472</td>
</tr>
<tr>
<td>Habitat for Humanity International</td>
<td>$467</td>
</tr>
<tr>
<td>Harvard University</td>
<td>$452</td>
</tr>
</tbody>
</table>

Source: The Chronicle of Philanthropy

8. Make a box-and-whisker for the data.
9. Write a brief description of the data distribution.

For Exercises 10–13, use the box-and-whisker plot at the right.

10. What is the range of the data?
11. What is the interquartile range of the data?
12. What fractional part of the data is less than 90?
13. What fractional part of the data is greater than 95?

Draw a box-and-whisker plot for each set of data.

14. 15, 8, 10, 1, 3, 2, 6, 5, 4, 27, 1
15. 20, 2, 12, 5, 4, 16, 17, 7, 6, 16, 5, 0, 5, 30
16. 4, 1, 1, 1, 10, 15, 4, 5, 27, 5, 14, 10, 6, 2, 2, 5, 8
17. 51, 27, 55, 54, 69, 60, 39, 46, 46, 53, 81, 23
18. 15.1, 9.0, 8.5, 5.8, 6.2, 8.5, 10.5, 11.5, 8.8, 7.6
19. 1.3, 1.2, 14, 1.8, 1.6, 5.7, 1.3, 3.7, 3.3, 2, 1.3, 1.3, 7.7, 8.5, 2.2

For Exercises 20–23, use the parallel box-and-whisker plot at the right.

20. Which set of data contains the least value?
21. Which set of data contains the greatest value?
22. Which set of data has the greatest interquartile range?
23. Which set of data has the greatest range?

Draw a parallel box-and-whisker plot for each pair of data. Compare the data.

24. A: 15, 17, 22, 28, 32, 40, 16, 24, 26, 38, 19
   B: 24, 32, 25, 27, 37, 29, 30, 30, 28, 31, 27
25. A: 50, 45, 47, 55, 51, 58, 49, 51, 51, 48, 47  
B: 40, 41, 48, 39, 41, 41, 38, 37, 35, 37, 45
26. A: 1.5, 3.8, 4.2, 3.5, 4.1, 4.4, 4.1, 4.0, 4.0, 3.9  
B: 6.8, 4.2, 7.6, 5.5, 12.2, 6.7, 7.1, 4.8
27. A: 4.4, 4.5, 4.6, 4.5, 4.4, 4.4, 4.1, 4.9, 2.9  
B: 5.1, 4.9, 4.2, 3.9, 4.5, 4.1, 4.3, 4.5, 5.2

**PROFESSIONAL SPORTS** For Exercises 28 and 29, use the table at the right.

28. Draw a box-and-whisker plot for the data.
29. What does the box-and-whisker plot tell about the data?

<table>
<thead>
<tr>
<th>Professional Sport</th>
<th>Average Length of Career (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bowling</td>
<td>17</td>
</tr>
<tr>
<td>Surfing</td>
<td>10</td>
</tr>
<tr>
<td>Hockey</td>
<td>5.5</td>
</tr>
<tr>
<td>Baseball</td>
<td>4.5</td>
</tr>
<tr>
<td>Basketball</td>
<td>4.5</td>
</tr>
<tr>
<td>Tennis</td>
<td>4</td>
</tr>
<tr>
<td>Football</td>
<td>3.5</td>
</tr>
<tr>
<td>Boxing</td>
<td>3.5</td>
</tr>
</tbody>
</table>

**RACING** For Exercises 30 and 31, use the following list of earnings in thousands from the November 2000 NAPA 500 NASCAR Race at the Atlanta Motor Speedway.

$181, $100, $98, $89, $76, $58, $60; $58; $55, $57, $54, $64, $44, $39, $66, $52, $56, $38, $56, $51, $49, $38, $50, $48, $48, $40, $36, $36, $39, $36, $47, $36, $47, $38, $35, $46, $35, $55, $46, $55, $45, $43, $43
Source: USA TODAY

30. Draw a box-and-whisker plot for the data. Identify any outliers.
31. Determine whether the top half of the data or the bottom half of the data are more dispersed. Explain.

**LIFE EXPECTANCY** For Exercises 32–35, use the box-and-whisker plot depicting the UNICEF life expectancy data for 171 countries.

32. Estimate the range and the interquartile range.
33. Determine whether the top half of the data or the bottom half of the data are more dispersed. Explain.
34. State three different intervals of ages that contain half the data.
35. Jamie claims that the number of data values is greater in the interval 54 years to 70 years than the number of data values in the interval 70 years to 74 years. Is Jamie correct? Explain.

**SOCCER** For Exercises 36–38, use the following list of top 50 lifetime scores for all players in Division 1 soccer leagues in the United States from 1922 to 1999.

253, 223, 193, 189, 152, 150, 138, 137, 135, 131, 131, 129, 128, 126, 124, 119, 118, 108, 107, 102, 101, 100, 96, 92, 87, 83, 82, 81, 80, 78, 78, 76, 74, 73, 73, 72, 71, 69, 68, 67, 65, 64, 63, 63, 63, 62, 61, 61
Source: www.internetsoccer.com

36. Draw a box-and-whisker plot for the data.
37. Create a histogram to represent the data.
38. Compare and contrast the box-and-whisker plot and the histogram.
39. **CRITICAL THINKING**  Write a set of data that could be represented by the box-and-whisker plot at the right.

40. **WRITING IN MATH**  Answer the question that was posed at the beginning of the lesson.

   **How are box-and-whisker plots used to display data?**
   Include the following in your answer:
   - a sample of a box-and-whisker plot showing what each part of the plot represents, and
   - a box-and-whisker plot representing data found in a newspaper or magazine.

---

**For Exercises 41 and 42, use the box-and-whisker plot below.**

41. What is the median of the data?
   - **A** 0  
   - **B** 10  
   - **C** 25  
   - **D** 45

42. Which interval represents 75% of the data?
   - **A** 0–25  
   - **B** 10–45  
   - **C** 25–50  
   - **D** 0–45

---

**Maintain Your Skills**

**Mixed Review**

For Exercises 43 and 44, use the following data.  13, 32, 45, 45, 54, 55, 58, 67, 82, 93

43. Find the range, median, lower quartile, upper quartile, and interquartile range of the data. Identify any outliers.  (**Lesson 13-4**)  
44. Create a histogram to represent the data.  (**Lesson 13-3**)  

Find each sum or difference.  (**Lesson 12-7**)  
45. \( \frac{3}{y - 3} - \frac{y}{y + 4} \)  
46. \( \frac{2}{r + 3} + \frac{3}{r - 2} \)  
47. \( \frac{w}{5w + 2} - \frac{4}{15w + 6} \)

Find each product. Assume that no denominator has a value of 0.  (**Lesson 12-3**)  
48. \( \frac{7a^2}{5} \cdot \frac{15}{14a} \)  
49. \( \frac{6r + 3}{r + 6} \cdot \frac{r^2 + 9r + 18}{2r + 1} \)

Solve each right triangle. State the side length to the nearest tenth and the angle measures to the nearest degree.  (**Lesson 11-7**)  
50. \( \angle C = 42^\circ, \quad \text{BC} = 22 \text{ in.} \)  
51. \( \angle B = 39^\circ, \quad \text{BC} = 12 \text{ m} \)  
52. \( \angle B = 46^\circ, \quad \text{AC} = 15 \text{ ft} \)

Solve each equation by completing the square. Approximate any irrational roots to the nearest tenth.  (**Lesson 10-3**)  
53. \( a^2 - 7a + 6 = 0 \)  
54. \( x^2 - 6x + 2 = 0 \)  
55. \( t^2 + 8t - 18 = 0 \)

Find each sum or difference.  (**Lesson 8-5**)  
56. \( (7p^2 - p - 7) - (p^2 + 11) \)  
57. \( (3a^2 - 8) + (5a^2 + 2a + 7) \)
Investigating Percentiles

When data are arranged in order from least to greatest, you can describe the data using percentiles. A **percentile** is the point below which a given percent of the data lies. For example, 50% of the data falls below the median. So the median is the 50th percentile for the data.

To determine a percentile, a cumulative frequency table can be used. In a **cumulative frequency table**, the frequencies are accumulated for each item.

**Collect the Data**

A student’s score on the SAT is one factor that some colleges consider when selecting applicants. The tables below show the raw scores from a sample math SAT test for 160 juniors in a particular school. For raw scores, the highest possible score is 800 and the lowest is 200.

<table>
<thead>
<tr>
<th>Math SAT Scores</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>200–300</td>
<td>2</td>
</tr>
<tr>
<td>300–400</td>
<td>19</td>
</tr>
<tr>
<td>400–500</td>
<td>44</td>
</tr>
<tr>
<td>500–600</td>
<td>55</td>
</tr>
<tr>
<td>600–700</td>
<td>32</td>
</tr>
<tr>
<td>700–800</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Math SAT Scores</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>200–300</td>
<td>2</td>
</tr>
<tr>
<td>300–400</td>
<td>19</td>
</tr>
<tr>
<td>400–500</td>
<td>44</td>
</tr>
<tr>
<td>500–600</td>
<td>55</td>
</tr>
<tr>
<td>600–700</td>
<td>32</td>
</tr>
<tr>
<td>700–800</td>
<td>8</td>
</tr>
</tbody>
</table>

The data in each table can be displayed in a histogram.

**Analyze the Data**

1. Examine the data in the two tables. Explain how the numbers in Column 3 of Table 2 are determined.

(continued on the next page)
2. Describe the similarities and differences between the two histograms.
3. Which histogram do you prefer for displaying these data? Explain your choice.

**Make a Conjecture**

Sometimes colleges are not interested in your raw score. They are interested in the percentile. Your percentile indicates what percent of all test-takers scored just as well or lower than you.

4. Use the histogram for Table 2. Place percentile labels on the vertical axis. For example, write 100% next to 160 and 0% next to 0. Now label 25%, 50%, and 75%. What numbers of students correspond to 25%, 50%, and 75%?

5. Suppose a college is interested in students with scores in the 90th percentile. Using the histogram, move up along the vertical axis to the 90th percentile. Then move right on the horizontal axis to find the score. What is an estimate for the score that represents the 90th percentile?

6. For a more accurate answer, use a proportion to find 90% of the total number of students. (Recall that the total number of students is 160.)

7. If a student is to be in the 90th percentile, in what interval will the score lie?

**Extend the Activity**

For Exercises 8–10, use the following information.
The weights of 45 babies born at a particular hospital during the month of January are shown below.

<table>
<thead>
<tr>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 lb 1 oz</td>
</tr>
<tr>
<td>8 lb 2 oz</td>
</tr>
<tr>
<td>7 lb 1 oz</td>
</tr>
<tr>
<td>5 lb 0 oz</td>
</tr>
<tr>
<td>7 lb 2 oz</td>
</tr>
<tr>
<td>3 lb 0 oz</td>
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<tr>
<td>8 lb 0 oz</td>
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<td>4 lb 4 oz</td>
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<td>11 lb 2 oz</td>
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<tr>
<td>6 lb 1 oz</td>
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<tr>
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<tr>
<td>7 lb 8 oz</td>
</tr>
<tr>
<td>8 lb 1 oz</td>
</tr>
<tr>
<td>7 lb 8 oz</td>
</tr>
<tr>
<td>7 lb 10 oz</td>
</tr>
</tbody>
</table>

8. Make a cumulative frequency table for the data.
9. Make a cumulative frequency histogram for the data.
10. Find the weight for a baby in the 80th percentile.
Chapter 13
Study Guide and Review

Vocabulary and Concept Check

<table>
<thead>
<tr>
<th>biased sample (p. 709)</th>
<th>histogram (p. 722)</th>
<th>range (p. 731)</th>
</tr>
</thead>
<tbody>
<tr>
<td>box-and-whisker plot (p. 737)</td>
<td>interquartile range (p. 732)</td>
<td>row (p. 715)</td>
</tr>
<tr>
<td>census (p. 708)</td>
<td>lower quartile (p. 732)</td>
<td>sample (p. 708)</td>
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<tr>
<td>column (p. 715)</td>
<td>matrix (p. 715)</td>
<td>scalar multiplication (p. 717)</td>
</tr>
<tr>
<td>convenience sample (p. 709)</td>
<td>measurement classes (p. 722)</td>
<td>simple random sample (p. 708)</td>
</tr>
<tr>
<td>dimensions (p. 715)</td>
<td>measures of variation (p. 731)</td>
<td>stratified random sample (p. 708)</td>
</tr>
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<td>element (p. 715)</td>
<td>outlier (p. 733)</td>
<td>systematic random sample (p. 708)</td>
</tr>
<tr>
<td>extreme value (p. 737)</td>
<td>population (p. 708)</td>
<td>upper quartile (p. 732)</td>
</tr>
<tr>
<td>frequency (p. 722)</td>
<td>quartiles (p. 732)</td>
<td>voluntary response sample (p. 709)</td>
</tr>
<tr>
<td>frequency table (p. 722)</td>
<td>random sample (p. 708)</td>
<td></td>
</tr>
</tbody>
</table>

Choose the correct term from the list above that best completes each statement.

1. A(n) ______ is a sample that is as likely to be chosen as any other from the population.
2. Measures that describe the spread of the values in a set of data are called ______.
3. Each ______ separates a data set into four sets with equal number of members.
4. In a(n) ______, the items are selected according to a specified time or item interval.
5. A(n) ______ has a systematic error within it so that certain populations are favored.
6. In a(n) ______, the population is first divided into similar, nonoverlapping groups.
7. The ______ is found by subtracting the lower quartile from the upper quartile.
8. A(n) ______ involves only those who want to participate in the sampling.
9. An extreme value that is much less or greater than the rest of the data is a(n) ______.
10. The ______ is the difference between the greatest and least values of a data set.

Lesson-by-Lesson Review

13-1 Sampling and Bias

Concept Summary
- Samples are used to represent a larger group called a population.
- Simple random sample, stratified random sample, and systematic random sample are types of unbiased, or random, samples.
- Convenience sample and voluntary response sample are types of biased samples.

Example
GOVERNMENT To determine whether voters support a new trade agreement, 5 people from the list of registered voters in each state and the District of Columbia are selected at random. Identify the sample, suggest a population from which it was selected, and state whether the sample is unbiased or biased. If unbiased, classify the sample as simple, stratified, or systematic. If biased, classify as convenience or voluntary response.

Since \(5 \times 51 = 255\), the sample is 255 registered voters in the United States. The population is all of the registered voters in the United States.

The sample is unbiased. It is an example of a stratified random sample.
**Introduction to Matrices**

**Concept Summary**
- A matrix can be used to organize data and make data analysis more convenient.
- Equal matrices must have the same dimensions and corresponding elements are equal.
- Matrices with the same dimensions can be added or subtracted.
- Each element of a matrix can be multiplied by a number called a scalar.

**Example**

If \( R = \begin{bmatrix} 2 & 2 \\ -1 & 3 \end{bmatrix} \) \( S = \begin{bmatrix} -1 & 3 \\ 0 & 1 \end{bmatrix} \) and \( T = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \), find each sum. If it does not exist, write impossible.

a. \( R + S \)

\[
R + S = \begin{bmatrix} 2 & 2 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 + (-1) & 2 + 3 \\ -1 + 0 & 3 + 1 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ -1 & 4 \end{bmatrix}
\]

b. \( S + T \)

\[
S + T = \begin{bmatrix} -1 & 3 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 -1 & 3 + 0 \\ 0 + 0 & 1 + 1 \end{bmatrix} \]

Since \( S \) is a \( 2 \times 2 \) matrix and \( T \) is a \( 2 \times 1 \) matrix, the matrices do not have the same dimensions. Therefore, it is impossible to add these matrices.

**Exercises**

If \( A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 0 & 4 \\ -1 & -1 & 3 \end{bmatrix} \), \( B = \begin{bmatrix} 1 & 1 & -3 \\ 2 & 3 & -1 \\ -1 & -2 & 0 \end{bmatrix} \), \( C = \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} \), and \( D = \begin{bmatrix} 2 & 1 \\ -2 & 0 \end{bmatrix} \), find each sum, difference, or product. If the sum or difference does not exist, write impossible. See Examples 3 and 4 on pages 716 and 717.

13. \( A + B \) 14. \( 3B \) 15. \(-2D\) 16. \( C - D \) 17. \( C + D \) 18. \( B + C \) 19. \( 5A \) 20. \( A - D \) 21. \( C + 3D \) 22. \( 2A - B \)

**Histograms**

**Concept Summary**
- A histogram can illustrate the information in a frequency table.
- The distribution of the data can be determined from a histogram.
Create a histogram to represent the following high temperatures in twenty states.

118 122 117 105 114 115 122 102 103 110
110 112 106 109 100 103 110 108 111 102

Since the temperatures range from 100 to 122, use measurement classes from 100 to 125 with 5 degree intervals. First create a frequency table and then draw the histogram.

<table>
<thead>
<tr>
<th>Temperature Intervals</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 ≤ d &lt; 105</td>
<td></td>
<td></td>
</tr>
<tr>
<td>105 ≤ d &lt; 110</td>
<td></td>
<td></td>
</tr>
<tr>
<td>110 ≤ d &lt; 115</td>
<td></td>
<td></td>
</tr>
<tr>
<td>115 ≤ d &lt; 120</td>
<td></td>
<td></td>
</tr>
<tr>
<td>120 ≤ d &lt; 125</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Exercises

Create a histogram to represent each data set. See Example 3 on page 724.

23. the number of cellular minutes used last month by employees of a company

<table>
<thead>
<tr>
<th>122</th>
<th>150</th>
<th>110</th>
<th>290</th>
<th>145</th>
<th>330</th>
<th>300</th>
<th>210</th>
<th>95</th>
<th>101</th>
<th>106</th>
<th>289</th>
<th>219</th>
</tr>
</thead>
<tbody>
<tr>
<td>105</td>
<td>302</td>
<td>29</td>
<td>288</td>
<td>154</td>
<td>235</td>
<td>168</td>
<td>55</td>
<td>84</td>
<td>92</td>
<td>175</td>
<td>180</td>
<td></td>
</tr>
</tbody>
</table>

24. the number of cups of coffee consumed per customer at a snack shop between 6 A.M. and 8 A.M.

<table>
<thead>
<tr>
<th>0</th>
<th>2</th>
<th>0</th>
<th>2</th>
<th>1</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>0</th>
<th>2</th>
<th>2</th>
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<th>1</th>
<th>3</th>
<th>0</th>
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<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
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<td>3</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
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</table>

Measures of Variation

See pages 731–736.

Concept Summary

- The range of the data set is the difference between the greatest and the least values of the set and describes the spread of the data.
- The interquartile range is the difference between the upper and lower quartiles of a set of data. It is range of the middle half of the data.
- Outliers are values that are much less than or much greater than the rest of the data.

Example

Find the range, median, lower quartile, upper quartile, and interquartile range of the set of data below. Identify any outliers.

25, 20, 30, 24, 22, 26, 28, 29, 19

Order the set of data from least to greatest.

19 20 22 24 25 26 28 29 30

The range is 30 – 19 or 11.

The median is the middle number, 25.

The lower quartile is \(\frac{20 + 22}{2}\) or 21.

The upper quartile is \(\frac{28 + 29}{2}\) or 28.5.

The interquartile range is 28.5 – 21 or 7.5.

The outliers would be less than 21 – 1.5(7.5) or 9.75 and greater than 28.5 + 1.5(7.5) or 39.25. There are no outliers.
Exercises  Find the range, median, lower quartile, upper quartile, and interquartile range of each set of data. Identify any outliers.  
See Examples 1–3 on pages 731–733.

25. 30, 90, 40, 70, 50, 100, 80, 60
26. 3, 3.2, 45, 7, 2, 1, 3.4, 4, 5.3, 5, 78, 8, 21, 5
27. 85, 77, 58, 69, 62, 73, 55, 82, 67, 77, 59, 92, 75, 69, 76
28. 111.5, 70.7, 59.8, 68.6, 63.8, 254.8, 64.3, 82.3, 91.7, 88.9, 110.5, 77.1

13-5  Box-and-Whisker Plots

Concept Summary
• The vertical rule in the box of a box-and-whisker plot represents the median.
• The box of a box-and-whisker plot represents the interquartile range.
• The bullets at each end of a box-and-whisker plot are the extremes.
• Parallel box-and-whisker plots can be used to compare data.

Example
The following high temperatures (°F) were recorded during a two-week cold spell in St. Louis. Draw a box-and-whisker plot of the temperatures.

20 2 12 5 4 16 17
7 6 16 5 0 5 30

Order the data from least to greatest.

0 2 4 5 5 5 6 7 12 16 16 17 20 30

\[ Q_1 = \frac{6 + 7}{2} = 6.5 \]  \[ Q_2 = \frac{6 + 7}{2} = 6.5 \]

The interquartile range is \( 16 - 5 \) or 11. Check to see if there are any outliers.

\[ 5 - 1.5(11) = -11.5 \quad 16 + 1.5(11) = 32.5 \]

There are no outliers.

Exercises  Draw a box-and-whisker plot for each set of data.
See Example 1 on page 738.

29. The number of Calories in a serving of French fries at 13 restaurants are 250, 240, 220, 348, 199, 200, 125, 230, 274, 239, 212, 240, and 327.
30. Mrs. Lowery’s class has the following scores on their math tests.
60, 70, 70, 75, 80, 85, 85, 90, 95, 100
31. The average daily temperatures on a beach in Florida for each month of one year are 52.4, 55.2, 61.1, 67.0, 73.4, 79.1, 81.6, 81.2, 78.1, 69.8, 61.9, and 55.1.
Vocabulary and Concepts

In a matrix, identify each item described.
1. a vertical set of numbers
2. an entry in a matrix
3. a horizontal set of numbers
4. a constant multiplied by each element in the matrix
5. number of rows and columns

Skills and Applications

Identify the sample, suggest a population from which it was selected, and state whether it is unbiased (random) or biased. If unbiased, classify the sample as simple, stratified, or systematic. If biased, classify as convenience or voluntary response.

6. **DOGS** A veterinarian needs a sample of dogs in his kennel to be tested for fleas. She selects the first 5 dogs who run from the pen.

7. **LIBRARIES** A librarian wants to sample book titles checked out on Wednesday. He randomly chooses a book for each hour that the library is open.

If \( W = \begin{bmatrix} 2 & 3 & 1 \\ -1 & 0 & -1 \\ 2 & -2 & 0 \end{bmatrix} \), \( X = \begin{bmatrix} 4 & 2 & -1 \\ -2 & -2 & 0 \\ 0 & 1 & 2 \end{bmatrix} \), \( Y = \begin{bmatrix} 3 & -2 & 1 \\ -1 & -2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \) and \( Z = \begin{bmatrix} 3 & 1 & 6 \\ 4 & -1 & -1 \end{bmatrix} \) find each sum, difference, or product. If the sum or difference does not exist, write impossible.

8. \( W + X \)

9. \( Y - Z \)

10. \( 3X \)

11. \( -2Z \)

12. \( 2W - Z \)

13. \( Y - 2Z \)

Create a histogram to represent each data set.

14. 68 71 74 90 81 72 71 69 65 92 75 69 71 73 73

15. 68 74 80 83 70 80 74 74 70 71

16. 1055, 1075, 1095, 1125, 1005, 975, 1125, 1100, 1145, 1025, 1075

17. 0.4, 0.2, 0.5, 0.9, 0.3, 0.4, 0.5, 1.9, 0.5, 0.7, 0.8, 0.6, 0.2, 0.1, 0.4

Find the range, median, lower quartile, upper quartile, and interquartile range for each set of data. Identify any outliers.

18. 1, 3, 2, 2, 1, 9, 4, 6, 1, 10, 1, 4, 5, 10, 1, 3, 6

19. 14, 18, 9, 9, 12, 22, 16, 12, 14, 16, 15, 13, 9, 10, 11, 12

20. **STANDARDIZED TEST PRACTICE** Which box-and-whisker plot has the greatest interquartile range?

[Diagram of box-and-whisker plots]
1. Which equation represents a line perpendicular to the graph of \( y = 4x - 6? \)  
   \( \text{A} \) \( y = \frac{1}{4}x + \frac{1}{6} \)  
   \( \text{B} \) \( y = -\frac{1}{4}x + 2 \)  
   \( \text{C} \) \( y = -4x + 6 \)  
   \( \text{D} \) \( y = 4x + 6 \)  

2. A certain number is proportional to another number in the ratio 3:5. If 8 is subtracted from the sum of the numbers, the result is 32. What is the greater number?  
   \( \text{A} \) 15  
   \( \text{B} \) 25  
   \( \text{C} \) 35  
   \( \text{D} \) 40  

3. The expression \((x - 8)^2\) is equivalent to  
   \( \text{A} \) \( x^2 - 64 \)  
   \( \text{B} \) \( x^2 - 16x + 64 \)  
   \( \text{C} \) \( x^2 + 16x + 64 \)  
   \( \text{D} \) \( x^2 + 64 \)  

4. What is the least \( y \) value of the graph of \( y = x^2 - 4? \)  
   \( \text{A} \) 2  
   \( \text{B} \) 0  
   \( \text{C} \) -2  
   \( \text{D} \) -4  

5. The expression \(3\sqrt{72} - 3\sqrt{2}\) is equivalent to  
   \( \text{A} \) \( 3\sqrt{70} \)  
   \( \text{B} \) \( 3\sqrt{2} \)  
   \( \text{C} \) \( 15\sqrt{2} \)  
   \( \text{D} \) \( 5\sqrt{2} \)  

6. A 12-meter flagpole casts a 9-meter shadow. At the same time, the building next to it casts a 27-meter shadow. How tall is the building?  
   \( \text{A} \) 20.25 m  
   \( \text{B} \) 36 m  
   \( \text{C} \) 40 m  
   \( \text{D} \) 84 m  

7. Students are conducting a poll at Cedar Grove High School to determine whether to change the school colors. Which would be the best place to find an unbiased sample of students who represent the entire student body?  
   \( \text{A} \) a football practice  
   \( \text{B} \) a freshmen class party  
   \( \text{C} \) a Spanish class  
   \( \text{D} \) the cafeteria  

8. A Mars year is longer than an Earth year because Mars takes longer to orbit the Sun. The table shows a person’s age in both Earth years and Mars years. The data represent which kind of function?  
   \( \text{A} \) linear function  
   \( \text{B} \) quadratic function  
   \( \text{C} \) exponential function  
   \( \text{D} \) rational function  

9. Which car shows the least variation in miles per gallon?  
   \( \text{A} \) Car A  
   \( \text{B} \) Car B  
   \( \text{C} \) Car C  
   \( \text{D} \) Car D  

10. Which car model has the highest median miles per gallon?  
    \( \text{A} \) Car A  
    \( \text{B} \) Car B  
    \( \text{C} \) Car C  
    \( \text{D} \) Car D
Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

11. Factor $x^3 + 8x^2 + 16x$. (Lesson 9-3)

12. Solve $6x^2 + x - 2 = 0$ by factoring. (Lesson 9-4)

13. Simplify $\sqrt[4]{27}$. (Lesson 11-1)

14. Maren can do a job in 4 hours. Juliana can do the same job in 6 hours. Suppose Juliana works on the job for 2 hours and then is joined by Maren. Find the number of hours it will take both working together to finish the job. (Lesson 11-4)

15. The map below shows train tracks cutting across a grid of city streets. Newton Street is 1.5 miles apart from Olive Street, Olive Street is 1.5 miles apart from Pine Street, and the three streets are parallel to each other. If the distance between points A and B is 5 miles, then what is the distance in miles between points B and C? (Lesson 12-4)

Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

- A the quantity in Column A is greater,
- B the quantity in Column B is greater,
- C the two quantities are equal, or
- D the relationship cannot be determined from the information given.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>the sum of the next three terms of the arithmetic sequence $-250, 83, 416, \ldots$</td>
<td>the 67th term of the arithmetic sequence $-49, 2, 53, \ldots$</td>
</tr>
<tr>
<td>(Lesson 4-7)</td>
<td></td>
</tr>
</tbody>
</table>

17. the root of $y = -0.25x^2 + x - 1$ | the sum of the roots of $b = 3a^2 - 5a + 2$ |
| (Lesson 10-4) | |

18. the value of $x$ if $\frac{3x + 1}{14} = \frac{x}{4}$ | the value of $y$ if $\frac{45}{y} + 1 = 10$ |
| (Lesson 12-9) | |

Part 4 Open Ended

Record your answers on a sheet of paper. Show your work.

19. Construct a histogram for the following data. Use intervals of 40–50, 50–60, 60–70, 70–80, 80–90, and 90–100. (Lesson 13-3)

20. In Exercise 19, what percent of the data lies within the tallest bar? (Lesson 13-3)

21. Draw a box-and-whisker plot of the following test scores. (Lesson 13-5)

Test-Taking Tip

Questions 14 and 15 If a problem seems difficult, don’t panic. Reread the question slowly and carefully. Always ask yourself, “What have I been asked to find?” and, “What information will help me find the answer?”